Moses Brown School
Honors Precalculus

Name ____________________

Honors Elementary Functions

2011 Mid-Year Exam (2013 REVIEW COPY)

Jan. 24, 2012

180 pts. possible

This exam should include 4 pages, including (this) page of formulae. Point values are given for all problems. Please draw a box around your final answer for each problem. Take a deep breath and dig in. You'll be fine. Good luck!

Formulae

Simple interest

$A(t) = A_o(1 + r)^t$

Compound interest

$$A(t) = A_o\left(1+\frac{r}{n}\right)^{nt}$$

Half life

$$A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{k}}$$

Continuous growth

$A(t) = A_o e^{rt}$

Summary of discrete probabilities

A	$P(A) \in [0,1]$	Probability must be between zero and one.
not A **Subtraction rule**	$P(!A) = 1 - P(A)$	
A or B **Addition rule**	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = P(A) + P(B)$	 If A and B are mutually exclusive
A and B **Multiplication Rule**	$P(A \cap B) = P(A\|B)P(B)$ $P(A \cap B) = P(A)P(B)$	 If A and B are independent
A given B	$P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$	
Bayes' theorem	$P(A\|B) = \dfrac{P(B\|A) \cdot P(A)}{P(B)}$	Connects $P(B\|A)$ and $P(A\|B)$

Binomial coefficients: $$\binom{N}{r} = \frac{N!}{r!(N-r)!}$$

Sum of an arithmetic series $$S = \frac{n}{2}(a_1 + a_n)$$

Sum of a geometric series $$\sum_{i=0}^{n} a_1 r^n = a_1\left(\frac{1-r^n}{1-r}\right)$$

1. (10 pts., 5 pts. each) E*bbinhaus' Law of Forgetting* states that if a task is learned at a performance level P_o, then after a time interval t, the performance level will be given by

$$\log P = \log P_o - c \log(t + 1),$$

where c is a constant that depends on the type of task and t is measured in months.

(a) Solve for P.

(b) If your score on a history test is 90, what score would Ebbinghaus' formula predict you would get on a similar test after one year. (Assume that $c = 0.25$ for the subject of history.)

2. (10 pts.) The tallest known totem pole, carved from a single log 38.38 meters high, is in Beacon Hill Park, Victoria, B.C., Canada. If a lacrosse ball is dropped from this height and bounces back to 60% of its original height on each bounce, find the total vertical distance (ignore horizontal distance) traveled by the ball by the time it stops bouncing. Express the vertical distance traveled using a geometric series and find the sum of that infinite series to solve this problem.

3. (5 pts.) Given that $x+3i$ is a zero of $f(x) = x^4 - x^3 + 7x^2 - 9x - 18$, find all other zeros of f(x). Express your answers exactly.

4. (10 pts.) Sketch a graph of the function $f(x) = \dfrac{x-1}{x^2-1}$. Find and carefully label any asymptotes, holes, and/or x- or y-axis intercepts.

5. (5 pts.) Find $2x^4 + 11x^3 + 11x^2 - 3x + 4$ divided by $(x + 4)$.

6. (20 pts., 5 pts. each) Solve for x. Give exact answers only:

(a) $\ln(x - 3) = \ln(x + 4) - 6$

(b) $\dfrac{-x}{12}(x-12)-36 = x$

(c) $8e^x - e^{2x} = 16$

(d) $x-1 = \dfrac{9(x-1)}{x^2}$

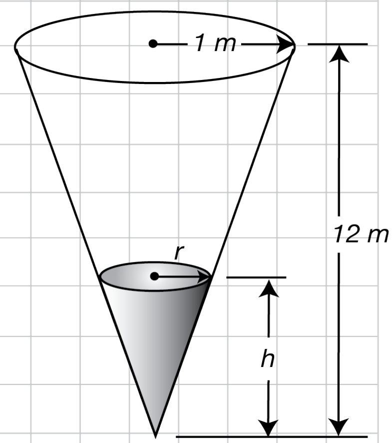
7. (10 pts.) A concert promoter has set the price of tickets to a concert at \$19.50, and plans to sell 30,000 seats. She notices in a trade journal that for concerts of this size, a rule of thumb is that for every \$1 ticket price increase, a promoter can expect to sell 1000 fewer seats. Given this information, find the price (to the penny) and number of seats sold that leads to maximum revenue for this concert.

8. (10 pts., 3,7) Five hours after the start of an experiment, there were 900 bacteria, and 5 hours after that there were 1020 bacteria. Write a formula for N(*t*), the number of bacteria *t* hours after the start of the experiment under each the assumptions (a and b) below. From each function, calculate the number of bacteria that were present at the start of the experiment, i.e. at $t = 0$.

   (a) assuming that N is a linear function of *t*

   (b) assuming that N is a continuous exponential function of *t.*

9. (10 pts.) A chemical tank in the shape of a right regular cone has a base radius of 1 meter, and a height of 12 meters, as shown. The tank is routinely filled with *1,3 butadiene* at a rate of 18 liters·min^{-1}. For chemical safety, it is crucial that the tank never overflow.



   (a) Find a formula for the volume V of the water in the trough as a function of the water level h. (Hint: remember similar triangles).

   (b) At what height, measured from the bottom, is the tank half full?

   Note: 1 m^3 = 1000 liters, $V_{cone} = \frac{1}{3}\pi r^2 h$

10. (12 pts., 3 pts. each) Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{2x}$. Find f(g(x)), g(f(x)), f(f(x)) and g(g(x)). Express the last function using rational exponents.

11. (13 pts.) An infectious disease begins to spread in a city of population 100,000. After t days, the number of people who have succumbed to the virus is modeled by the logistic function

$$v(t) = \frac{10{,}000}{5 + 1245e^{-0.97t}}$$

   (a) How many infected people are there initially (at time t = 0)?
   (b) Find the number of infected people after 14 days.
   (c) Sketch a graph of the function v(t) and describe its behavior in terms of the spread of the disease. Feel free to use your calculator as a guide.

12. (10 pts.) An open box with a square base is to have a volume of 12 ft^3.

   (a) Find a function with one independent variable that models the surface area of this box.

   (b) Find the box dimensions that *minimize* the amount of material used. (+1 pt. EC for using calculus to find the exact value)

13. (10 pts.) Find the inverse of the function $f(x) = x^2 - 4$. Write the domain and range of <u>both</u> $f(x)$ and $f^{-1}(x)$.

14. (15 pts., 5 pts. each) Simplify these expressions without using a calculator:

   (a) $\sqrt{\frac{1}{3}}\sqrt{-27}$ (b) $(3-\sqrt{-5})(1+\sqrt{-1})$ (c) $\frac{1-\sqrt{-1}}{1+\sqrt{-1}}$

15. (10 pts.) Suppose that a barrel contains many small plastic eggs. Some eggs are painted red and some are painted blue. 40% of the eggs in the bin contain pearls, and 60% contain nothing. 30% of eggs containing pearls are painted blue, and 10% of eggs containing nothing are painted blue. Calculate the probability that a blue egg contains a pearl.

16. (10 pts.) Sketch a graph of this function. Make sure to label and any asymptotes, x-intercepts or y-intercepts.

$$f(x) = = \frac{x^5+10}{x^4+8x^2+15} \quad .$$

17. (10 pts.) Take the parent function of all cubic functions and apply the following transformations in the order shown. After applying the transformations, express the function in standard form, $f(x) = Ax^3 + Bx^2 + Cx + D$.

   i. shift up by three units along the *y*-axis
   ii. shift left by two units along the *x*-axis
   iii. reflect across the *y*-axis