2014 MIDTERM REVIEW TOPICS:

Topics that we covered in the First Semester. The 2012/2013 Midterm and Midyear Review Problems are the best indicator of how much you know though. This is some helpful hints/ things to remember in each section. It is not in ANY way all encompassing of what we have learned. In addition please note that last year's midterm did not include sequences and series OR confidence intervals (but this exam will include both)

UNIT 1: FUNCTIONS

What functions are and are not.
Function Transformations (Shifts of the common functions)
Inverse of Functions
Domain and Range
Even/Odd Functions
Composition of Functions
Function Modeling Problems

Unit 2: Quadratic Functions

Vertex/Roots of Quadratics
Completing the Square
Deriving the Quadratic Formula
Complex Numbers
Quadratic Function Modeling

Unit 3: Polynomial Functions

Graphing Polynomials
Polynomial Long Division and Synthetic Division
Real Roots of Polynomials
Complex Roots of Polynomials
Fundamental Theorem of Algebra
Descartes' Rule of Signs *(count the number of sign changes – this is the max number of positive roots/plug in f(-x) and then count the sign changes again. This is the max number of negative roots.*
Upper/Lower Bounds *(all signs positive = an upper bound/alternating signs = lower bound)*

Unit 4: Rational Functions

Graphing Rational Functions (FAITS)
Vertical/Horizontal Asymptotes
Slant/Parabolic Asymptotes
Holes

**Piecewise Functions (not on midterm)

Unit 5: Statistics (focus on the part in bold)

Basic Stats (Discrete Stats)
Bayesian Statistics
Expected Value
Permutations/Combinations
The Normal Curve
Z-Values
Confidence Intervals for Means/Proportions

Won't be on the midterm
Hypothesis Testing for Means/Proportions

Unit 6: Exponential and Logarithmic Functions

Exponential Functions
e and the Natural Exponential function
Logarithms (graphing/laws/ etc.)
The Natural Logarithm (ln)
Solving Logs/Exponentials
Logs/Exponentials Modeling Problems
HALF LIFE
EXPONENTIAL GROWTH

UNIT 7: SEQUENCES AND SERIES

Sequences (standard, recursive)
Arithmetic and Geometric Sequences and Series
Partial and Infinite Sums
Expanding a binomial

UNIT 1: FUNCTIONS

What functions are and are not?

Functions are relations where for every input (x-value) there is exactly one corresponding output (y-value). Functions pass the vertical line test.

To test if the inverse of a function is ALSO a function, use the horizontal line test.

Functions Transformations (Shifts of the common functions)

How to transform a function: Let's take the cubic function for example. $f(x) = x^3$

Shift Left:	Shift Right:	Shift Up:	Shift Down:
$F(x) = (x + 2)^3$	$F(x) = (x - 2)^3$	$F(x) = x^3 + 2$	$F(x) = x^3 - 2$

Stretch Vertically:	Compress Vertically (flatten):	Flip over the x-axis
$F(x) = 2x^3$	$F(x) = \frac{1}{2}x^3$	$F(x) = -x^3$

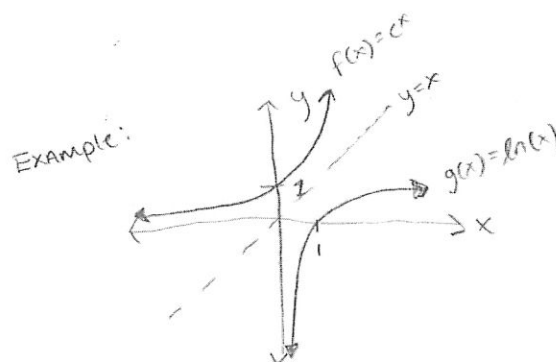| Flip over the y-axis	Stretch Horizontally	Compress Horizontally
$F(x) = (-x)^3$	$F(x) = \left(\frac{x}{0.5}\right)^3$	$f(x) = \left(\frac{x}{2}\right)^3$

Inverse of Functions:

Switch the x and y values and solve for y.

The function and its inverse are reflection over $y = x$.

EXAMPLE:

Domain and Range: Described using bracket notation.

The domain is the input values (x-values) that the function may take on.

Restrictions to the domain come from the denominator of a function not being allowed to be zero and from square roots needing to be greater than or equal to zero. In addition logs must be taken of values that are greater than zero.

The range is the output values (y-values) that the function may take on.

You may oftentimes find restrictions to the range by testing limits (i.e. what happens as x gets very large (approaches infinity) or very negative (i.e. approaches negative infinity).

Or if there is a restriction on the domain (such as D: [3, infinity)) Then to find the range, see what happens to y when x = 3 and what happens to y as x approaches infinity.

Also, just graph the function to visually see.

Even/Odd Functions

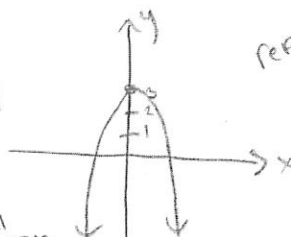*Even Functions: Functions which are symmetric about the y-axis.*

To prove it algebraically: $f(-x) = f(x)$ i.e. when you plug in (–x) for all x's in the function, the function is identical to the original function.

Example of an Even Function: $f(x) = -3x^2 + 4$

$$f(-x) = -3(-x)^2 + 4$$
$$f(-x) = -3x^2 + 4$$

same as the original function

reflection over y-axis.

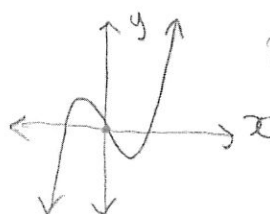*Odd Functions: Odd functions are symmetric about the origin.*

To prove it algebraically: $f(-x) = -f(x)$ i.e. when you plug in (–x) for all x's in the function, the function is the opposite of the original function.

Example of an Odd Function: $f(x) = 2x^3 - 4x$

$$f(-x) = 2(-x)^3 - 4(-x)$$
$$\boxed{f(-x) = -2x^3 + 4x}$$

opposite of the original function.

Reflected about the origin.

Composition of Functions

To compose two functions (i.e. f(g(x)) simply replace every x in the first function with the other function. Then simplify.

$$f(x) = x^2 + 2 \qquad g(x) = \sqrt{3x-4}$$

$$f(g(x)) = \left(\sqrt{3x-4}\right)^2 + 2$$

$$= 3x - 4 + 2$$

$$\boxed{f(g(x)) = 3x - 2}$$

Function Modeling Problems

See your practice midterm/tests/midterm review. (for example the conical tank is a function composition modeling problem).

Unit 2: Quadratic Functions

Vertex/Roots of Quadratics

1.) Vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

2.) Factor, Complete the Square or Use the Quadratic Formula to find the roots (zeros) of a quadratic function.

Completing the Square: Note you should know how to do this in case you need some EXACT answers (i.e. non decimals)

Quick Example Here:

$$\frac{2x^2 - 14x + 8 = 0}{2}$$

① Step 1: divide by the leading Coefficient.

$$x^2 - 7x + 4 = 0$$
$$-4 \qquad -4$$

② Step 2: put the c-term on the right side

$\left(\frac{b}{2}\right)^2$

$\left(\frac{-7}{2}\right)^2$

$(-3.5)^2$
$= 12.25$

$$x^2 - 7x + 12.25 = -4 + 12.25$$
$$\sqrt{(x-3.5)^2} = \sqrt{-8.25}$$

③ Step 3: add $\left(\frac{b}{2}\right)^2$ to both sides. (complete the square)

$$x - 3.5 = \pm\sqrt{8.25}\ i$$
$$+3.5 \qquad +3.5$$

Deriving the Quadratic Formula

$$\boxed{x = 3.5 \pm \sqrt{8.25}\ i}$$

Don't need to know for the midterm

Complex Numbers:

$i = \sqrt{-1}$
$i^2 = -1$

a + bi is standard form of a complex number. (example: 5 + 3i)

Quadratic Function Modeling:

See notes and 2012 midterm (for example the revenue problem).

Unit 3: Polynomial Functions

Graphing Polynomials

1.) Determine End Behavior (look at the degree and leading coefficient)
2.) Find the x-intercepts (zeros) by factoring and finding all the roots
3.) Make a Table and determine what happens in between and after each x-intercept.

Polynomial Long Division and Synthetic Division

See the 2012 Midterm Solutions for examples of this and your notes.

Real Roots/ Complex Roots of Polynomials

1.) Try to factor by grouping, factor out a GCF, or see if it is a sum or difference of cubes etc.

2.) If you cannot factor by grouping, then use P/Q to find all of the possible rational roots of the polynomial.

3.) To determine if one of these roots is a factor of the polynomial, use synthetic division. If there is NO REMAINDER than it is a factor.

IMPORTANT: For example if I tried -2 and there was no remainder than the factor would be (x + 2), since -2 was a zero!

4.) Keep going until you can factor a quadratic etc. by hand or perhaps you will find some imaginary solutions as well from factoring.

Fundamental Theorem of Algebra

If you have a nth degree polynomial, the polynomial has EXACTLY n roots (real and complex.

Remember that complex roots always come in conjugate pairs (i.e. (x + 3i) and (x – 3i)

SOME USEFUL STRATEGIES TO MAKE P/Q EASIER:

Descartes' Rule of Signs

$$f(x) = 5x^4 - 3x^3 + 2x^2 + 3x + 4$$

Count the number of sign changes – this is the max number of positive roots.
Plug in f(-x) and then count the sign changes. This is the max number of negative roots.

2 sign changes.
So there are 2 or 0 positive roots.

$$f(-x) = 5(-x)^4 - 3(-x)^3 + 2(-x)^2 + 3(-x) + 4$$
$$= 5x^4 + 3x^3 + 2x^2 - 3x + 4$$

2 sign changes. So 2 or 0 neg. roots.

Upper/Lower Bounds

When testing possible factors (P/Q) using synthetic division:
    IF all signs positive in the answer it is an upper bound (i.e. no need to test possible zeros higher)
    IF all signs alternating (i.e. go from + - + -) it is a lower bound (i.e. no need to test possible zeros lower)

Unit 4: Rational Functions

Graphing Rational Functions (FAITS)

Factor
Asymptotes
Intercepts
Table of Values
Sketch the graph

$n < m$: Horizontal Asymptote at $y=0$

$n = m$: horizontal asymptote at the fraction of the leading coefficients. $\left(f(x) = \frac{2x+3}{3x-7} \right)$

$n > m$: Slant asymptote ($n>m$ by 1) or Parabolic asymptote ($n>m$ by 2) $\boxed{i.e \ y=\frac{2}{3}}$

(Do LONG DIVISION and "Throw-out" the remainder)

FACTOR:
First factor wherever possible (this will help identify holes and the x intercepts.)

Asymptotes/Holes

Vertical Asymptotes come from the denominator being zero.

Horizontal Asymptotes: $n < m$ than a horizontal asymptote at $y = 0$

If $n = m$ than a horizontal asymptote at the fraction of the leading coefficients (numerator coefficient/denominator coefficient).

Slant Asymptote: $n>m$ by 1 degree than a slant asymptote. (Use polynomial long division or synthetic division to divide the numerator by the denominator. Throw out the remainder and what is left is the equation of the slant asymptote.

Parabolic Asymptote: $n>m$ by 2 degrees than there will be a parabolic asymptote. (Use polynomial long division or synthetic division to divide the numerator by the denominator. Throw out the remainder and what is left is the equation of the parabolic asymptote.

Holes: If you can eliminate a factor (such as (x + 2)) from the numerator and denominator, this will create a hole. This is because there will now not be a vertical asymptote at x = -2 but there will be a hole there. DENOTE this on the graph with an OPEN circle.

INTERCEPTS

X-Intercept(s): When the numerator is equal to zero. This is why you factor.

Y-Intercept(s): Plug in zero for x and solve for y.

TABLE OF VALUES:

Create a table of values including all intercepts. Then pick points to the left and right of vertical asymptotes. In addition, it is sometimes helpful to pick other points to see if your function crosses the horizontal or slant asymptotes.

SKETCH THE GRAPH: Sketch a graph showing all features listed above.

from 5b on
the Midyear Review.

FAITS

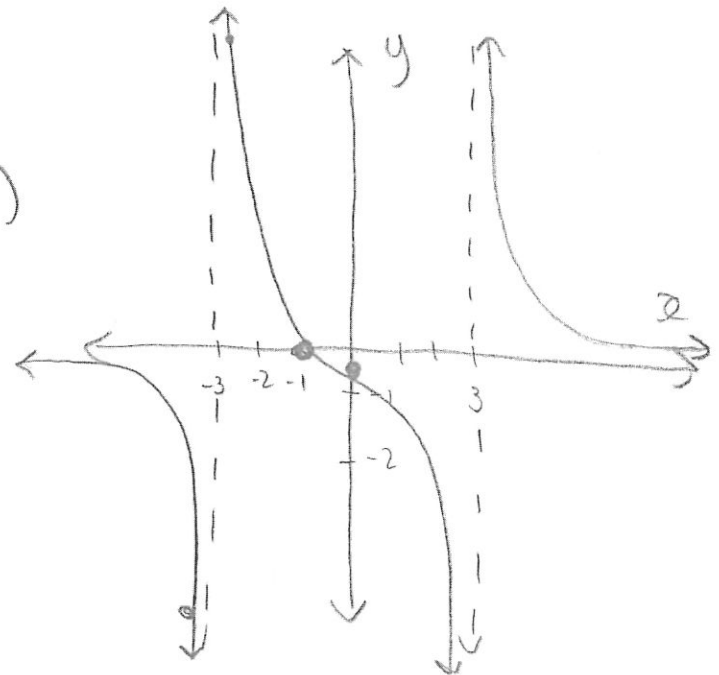$$f(x) = \frac{x + 1}{x^2 - 9}$$

Sketch

FACTOR: $\dfrac{x + 1}{(x+3)(x-3)}$

Asymptotes:

HA: $y = 0$ $(n < m)$

VA: $x = 3$ $x = -3$

Slant/Parabolic: none

holes: none.

Intercepts:

x-int: $x + 1 = 0$

$x = -1$

$\boxed{(-1, 0)}$

Y-int. $\dfrac{0 + 1}{0 - 9} = -\dfrac{1}{9}$ $\boxed{\left(0, -\dfrac{1}{9}\right)}$

Table

x	y
-1	0
0	$-\frac{1}{9}$
-3.1	$-\infty$
-2.9	∞
2.9	$-\infty$
3.1	∞

Unit 5: Statistics

Mainly focus on Bayes Theorem and Permutations/Combinations for the Midterm.

As well as conf. Intervals

Basic Stats (Discrete Stats)

Bayesian Statistics: Conditional Probability.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Expected Value (~~don't worry about this for the midterm – though it's extremely useful~~)

Permutations/Combinations

i.e. how many ways are there to arrange the letters in the word TRIGONOMETRY?

OR how many ways are there to select a group of 15 people from a group of 50 applicants?

Combinations $50 \, nCr \, 15$
(a group is a group)

The Normal Curve
Z-Values

Won't be on the midterm
Confidence Intervals for Means/Proportions
Hypothesis Testing for Means/Proportions

Unit 6: Exponential and Logarithmic Functions

We did UNIT 6 and 7 very recently so I won't make a review sheet on these, but as you saw on the 2012/2013 midterm, you should be very comfortable with solving logs/exponentials and expect to see a modeling problem or 2 as well.

Exponential Functions
e and the Natural Exponential function
Logarithms (graphing/laws/ etc.)
The Natural Logarithm (ln)
Solving Logs/Exponentials
Logs/Exponentials Modeling Problems
    HALF LIFE, EXPONENTIAL GROWTH

UNIT 7: SEQUENCES AND SERIES

See your recent quiz. All formulas will be given

Sequences (standard, recursive)
Arithmetic and Geometric Sequences and Series
Partial and Infinite Sums
Expanding a binomial