2.4 EXERCISES

CONCEPTS

1. If you travel 100 miles in two hours, then your average speed for the trip is

$$\text{average speed} = \frac{\quad}{\quad} = \_\_\_\_\_$$

2. The average rate of change of a function f between $x = a$ and $x = b$ is

$$\text{average rate of change} = \frac{\quad}{\quad}$$

3. The average rate of change of the function $f(x) = x^2$ between $x = 1$ and $x = 5$ is

$$\text{average rate of change} = \frac{\quad}{\quad} = \_\_\_\_\_$$

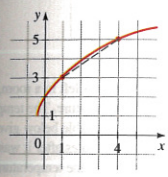
4. (a) The average rate of change of a function f between $x = a$ and $x = b$ is the slope of the _____ line between $(a, f(a))$ and $(b, f(b))$.

(b) The average rate of change of the linear function $f(x) = 3x + 5$ between any two points is _____.

SKILLS

5–8 ■ The graph of a function is given. Determine the average rate of change of the function between the indicated points on the graph.

5.



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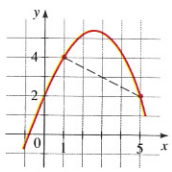

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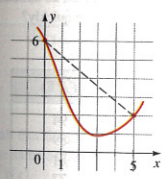

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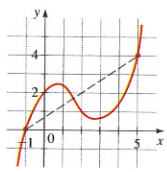


9–20 ■ A function is given. Determine the average rate of change of the function between the given values of the variable.

9. $f(x) = 3x - 2;\quad x = 2, x = 3$

10. $g(x) = 5 + \frac{1}{2}x;\quad x = 1, x = 5$

11. $h(t) = t^2 + 2t;\quad t = -1, t = 4$

12. $f(z) = 1 - 3z^2;\quad z = -2, z = 0$

13. $f(x) = x^3 - 4x^2;\quad x = 0, x = 10$

14. $f(x) = x + x^4;\quad x = -1, x = 3$

15. $f(x) = 3x^2;\quad x = 2, x = 2 + h$

16. $f(x) = 4 - x^2;\quad x = 1, x = 1 + h$

17. $g(x) = \dfrac{1}{x};\quad x = 1, x = a$

18. $g(x) = \dfrac{2}{x + 1};\quad x = 0, x = h$

19. $f(t) = \dfrac{2}{t};\quad t = a, t = a + h$

20. $f(t) = \sqrt{t};\quad t = a, t = a + h$

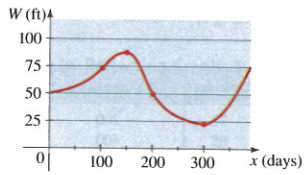
21–22 ■ A linear function is given. **(a)** Find the average rate of change of the function between $x = a$ and $x = a + h$. **(b)** Show that the average rate of change is the same as the slope of the line.

21. $f(x) = \frac{1}{2}x + 3$

22. $g(x) = -4x + 2$

APPLICATIONS

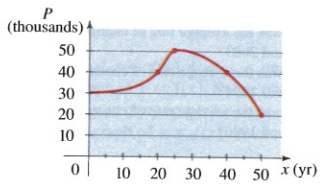
23. **Changing Water Levels** The graph shows the depth of water W in a reservoir over a one-year period as a function of the number of days x since the beginning of the year. What was the average rate of change of W between $x = 100$ and $x = 200$?



24. **Population Growth and Decline** The graph shows the population P in a small industrial city from 1950 to 2000. The variable x represents the number of years since 1950.

(a) What was the average rate of change of P between $x = 20$ and $x = 40$?

(b) Interpret the value of the average rate of change that you found in part (a).



DRILL QUESTION

If $f(t) = t^2 - |3t|$, what is the average rate of change between $t = -3$ and $t = -1$?

Answer

-1

25. Population Growth and Decline The table gives the population in a small coastal community for the period 1997–2006. Figures shown are for January 1 in each year.

(a) What was the average rate of change of population between 1998 and 2001?

(b) What was the average rate of change of population between 2002 and 2004?

(c) For what period of time was the population increasing?

(d) For what period of time was the population decreasing?

Year	Population
1997	624
1998	856
1999	1,336
2000	1,578
2001	1,591
2002	1,483
2003	994
2004	826
2005	801
2006	745

26. Running Speed A man is running around a circular track that is 200 m in circumference. An observer uses a stopwatch to record the runner's time at the end of each lap, obtaining the data in the following table.

(a) What was the man's average speed (rate) between 68 s and 152 s?

(b) What was the man's average speed between 263 s and 412 s?

(c) Calculate the man's speed for each lap. Is he slowing down, speeding up, or neither?

Time (s)	Distance (m)
32	200
68	400
108	600
152	800
203	1000
263	1200
335	1400
412	1600

27. CD Player Sales The table shows the number of CD players sold in a small electronics store in the years 1993–2003.

(a) What was the average rate of change of sales between 1993 and 2003?

(b) What was the average rate of change of sales between 1993 and 1994?

(c) What was the average rate of change of sales between 1994 and 1996?

(d) Between which two successive years did CD pla *increase* most quickly? *Decrease* most quickly?

Year	CD players sold
1993	512
1994	520
1995	413
1996	410
1997	468
1998	510
1999	590
2000	607
2001	732
2002	612
2003	584

28. Book Collection Between 1980 and 2000, a rare collector purchased books for his collection at the rate books per year. Use this information to complete the f table. (Note that not every year is given in the table.)

Year	Number of books
1980	420
1981	460
1982	
1985	
1990	
1992	
1995	
1997	
1998	
1999	
2000	1220

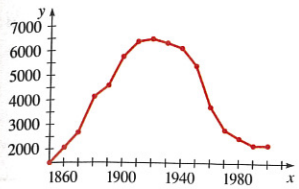
29. Cooling Soup When a bowl of hot soup is left in a r the soup eventually cools down to room temperature. The ature T of the soup is a function of time t. The table below the temperature (in °F) of a bowl of soup t minutes after it on the table. Find the average rate of change of the tempe the soup over the first 20 minutes and over the next 20 mi During which interval did the soup cool off more quickly?

t (min)	T (°F)	t (min)	T (°F)
0	200	35	94
5	172	40	89
10	150	50	81
15	133	60	77
20	119	90	72
25	108	120	70
30	100	150	70

30. Farms in the United States The graph gives the number of farms in the United States from 1850 to 2000.

(a) Estimate the average rate of change in the number of farms between (i) 1860 and 1890 and (ii) 1950 and 1970.

(b) In which decade did the number of farms experience the greatest average rate of decline?

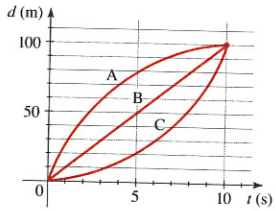


DISCOVERY ▪ DISCUSSION ▪ WRITING

31. 100-Meter Race A 100-m race ends in a three-way tie for first place. The graph at the top of the next column shows distance as a function of time for each of the three winners.

(a) Find the average speed for each winner.

(b) Describe the differences between the ways in which the three runners ran the race.



32. Linear Functions Have Constant Rate of Change If $f(x) = mx + b$ is a linear function, then the average rate of change of f between any two real numbers x_1 and x_2 is

$$\text{average rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Calculate this average rate of change to show that it is the same as the slope m.

33. Functions with Constant Rate of Change Are Linear If the function f has the same average rate of change c between any two points, then for the points a and x we have

$$c = \frac{f(x) - f(a)}{x - a}$$

Rearrange this expression to show that

$$f(x) = cx + (f(a) - ca)$$

and conclude that f is a linear function.

2.5 TRANSFORMATIONS OF FUNCTIONS

Vertical Shifting ▶ Horizontal Shifting ▶ Reflecting Graphs ▶ Vertical Stretching and Shrinking ▶ Horizontal Stretching and Shrinking ▶ Even and Odd Functions

SUGGESTED TIME AND EMPHASIS
1 class, essential material

In this section we study how certain transformations of a function affect its graph. This will give us a better understanding of how to graph functions. The transformations that we study are shifting, reflecting, and stretching.

▼ Vertical Shifting

Adding a constant to a function shifts its graph vertically: upward if the constant is positive and downward if it is negative.

In general, suppose we know the graph of $y = f(x)$. How do we obtain from it the graphs of

$$y = f(x) + c \quad \text{and} \quad y = f(x) - c \qquad (c > 0)$$

Recall that the graph of the function f is the same as the graph of the equation $y = f(x)$.

The y-coordinate of each point on the graph of $y = f(x) + c$ is c units above the y-coordinate of the corresponding point on the graph of $y = f(x)$. So we obtain the graph of $y = f(x) + c$ simply by shifting the graph of $y = f(x)$ upward c units. Similarly, we obtain the graph of $y = f(x) - c$ by shifting the graph of $y = f(x)$ downward c units.

TEXT QUESTION
What is the difference between a vertical stretch and a vertical shift?

Answer
A vertical stretch extends the graph in the vertical direction, changing its shape. A vertical shift simply moves the graph in the vertical direction, preserving its shape.