8. (a) What is an even function? What symmetry does its graph possess? Give an example of an even function.

(b) What is an odd function? What symmetry does its graph possess? Give an example of an odd function.

9. What does it mean to say that $f(3)$ is a local maximum value of f?

10. Suppose that f has domain A and g has domain B.

(a) What is the domain of $f + g$?

(b) What is the domain of fg?

(c) What is the domain of f/g?

11. How is the composite function $f \circ g$ defined?

12. (a) What is a one-to-one function?

(b) How can you tell from the graph of a function whether it is one-to-one?

(c) Suppose f is a one-to-one function with domain A and range B. How is the inverse function f^{-1} defined? What is the domain of f^{-1}? What is the range of f^{-1}?

(d) If you are given a formula for f, how do you find a formula for f^{-1}?

(e) If you are given the graph of f, how do you find the graph of f^{-1}?

■ EXERCISES

1–2 ■ A verbal description of a function f is given. Find a formula that expresses f in function notation.

1. "Square, then subtract 5."

2. "Divide by 2, then add 9."

3–4 ■ A formula for a function f is given. Give a verbal description of the function.

3. $f(x) = 3(x + 10)$

4. $f(x) = \sqrt{6x - 10}$

5–6 ■ Complete the table of values for the given function.

5. $g(x) = x^2 - 4x$

x	$g(x)$
-1	
0	
1	
2	
3	

6. $h(x) = 3x^2 + 2x - 5$

| x	$h(x)$
-2	
-1	
0	
1	
2	

7. A publisher estimates that the cost $C(x)$ of printing a run of x copies of a certain mathematics textbook is given by the function $C(x) = 5000 + 30x - 0.001x^2$.

(a) Find $C(1000)$ and $C(10,000)$.

(b) What do your answers in part (a) represent?

(c) Find $C(0)$. What does this number represent?

8. Reynalda works as a salesperson in the electronics division of a department store. She earns a base weekly salary plus a commission based on the retail price of the goods she has sold. If she sells x dollars worth of goods in a week, her earnings for that week are given by the function $E(x) = 400 + 0.03x$.

(a) Find $E(2000)$ and $E(15,000)$.

(b) What do your answers in part (a) represent?

(c) Find $E(0)$. What does this number represent?

(d) From the formula for E, determine what percentage Reynalda earns on the goods that she sells.

9. If $f(x) = x^2 - 4x + 6$, find $f(0)$, $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(x + 1)$, $f(2x)$, and $2f(x) - 2$.

10. If $f(x) = 4 - \sqrt{3x - 6}$, find $f(5)$, $f(9)$, $f(a + 2)$, $f(-x)$, $f(x^2)$, and $[f(x)]^2$.

11. Which of the following figures are graphs of functions? Which of the functions are one-to-one?

(a)

(b)

(c)

(d)
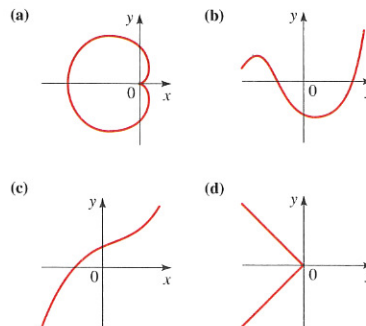
12. The graph of a function f is given.

(a) Find $f(-2)$ and $f(2)$.

(b) Find the domain of f.

(c) Find the range of f.

(d) On what intervals is f increasing? On what intervals is f decreasing?

(e) What are the local maximum values of f?

(f) Is f one-to-one?
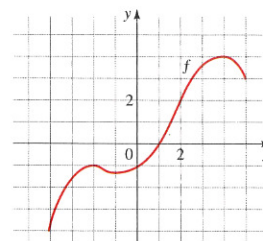
13–14 ■ Find the domain and range of the function.

13. $f(x) = \sqrt{x + 3}$

14. $F(t) = t^2 + 2t + 5$

15–22 ■ Find the domain of the function.

15. $f(x) = 7x + 15$

16. $f(x) = \dfrac{2x + 1}{2x - 1}$

17. $f(x) = \sqrt{x + 4}$

18. $f(x) = 3x - \dfrac{2}{\sqrt{x + 1}}$

19. $f(x) = \dfrac{1}{x} + \dfrac{1}{x + 1} + \dfrac{1}{x + 2}$

20. $g(x) = \dfrac{2x^2 + 5x + 3}{2x^2 - 5x - 3}$

21. $h(x) = \sqrt{4 - x} + \sqrt{x^2 - 1}$

22. $f(x) = \dfrac{\sqrt[3]{2x + 1}}{\sqrt[3]{2x + 2}}$

23–40 ■ Sketch the graph of the function.

23. $f(x) = 1 - 2x$

24. $f(x) = \frac{1}{3}(x - 5), \ 2 \le x \le 8$

25. $f(t) = 1 - \frac{1}{2}t^2$

26. $g(t) = t^2 - 2t$

27. $f(x) = x^2 - 6x + 6$

28. $f(x) = 3 - 8x - 2x^2$

29. $g(x) = 1 - \sqrt{x}$

30. $g(x) = -|x|$

31. $h(x) = \frac{1}{2}x^3$

32. $h(x) = \sqrt{x + 3}$

33. $h(x) = \sqrt[3]{x}$

34. $H(x) = x^3 - 3x^2$

35. $g(x) = \dfrac{1}{x^2}$

36. $G(x) = \dfrac{1}{(x - 3)^2}$

37. $f(x) = \begin{cases} 1 - x & \text{if } x < 0 \\ 1 & \text{if } x \ge 0 \end{cases}$

38. $f(x) = \begin{cases} 1 - 2x & \text{if } x \le 0 \\ 2x - 1 & \text{if } x > 0 \end{cases}$

39. $f(x) = \begin{cases} x + 6 & \text{if } x < -2 \\ x^2 & \text{if } x \ge -2 \end{cases}$

40. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x < 2 \\ 1 & \text{if } x \ge 2 \end{cases}$

41–44 ■ Determine whether the equation defines y as a function of x.

41. $x + y^2 = 14$

42. $3x - \sqrt{y} = 8$

43. $x^3 - y^3 = 27$

44. $2x = y^4 - 16$

45. Determine which viewing rectangle produces the most appropriate graph of the function
$$f(x) = 6x^3 - 15x^2 + 4x - 1$$
(i) $[-2, 2]$ by $[-2, 2]$ (ii) $[-8, 8]$ by $[-8, 8]$
(iii) $[-4, 4]$ by $[-12, 12]$ (iv) $[-100, 100]$ by $[-100, 100]$

46. Determine which viewing rectangle produces the most appropriate graph of the function $f(x) = \sqrt{100 - x^3}$.
(i) $[-4, 4]$ by $[-4, 4]$
(ii) $[-10, 10]$ by $[-10, 10]$
(iii) $[-10, 10]$ by $[-10, 40]$
(iv) $[-100, 100]$ by $[-100, 100]$

47–50 ■ Draw the graph of the function in an appropriate viewing rectangle.

47. $f(x) = x^2 + 25x + 173$

48. $f(x) = 1.1x^3 - 9.6x^2 - 1.4x + 3.2$

49. $f(x) = \dfrac{x}{\sqrt{x^2 + 16}}$

50. $f(x) = |x(x + 2)(x + 4)|$

51. Find, approximately, the domain of the function
$$f(x) = \sqrt{x^3 - 4x + 1}$$

52. Find, approximately, the range of the function
$$f(x) = x^4 - x^3 + x^2 + 3x - 6$$

53–54 ■ Draw a graph of the function f, and determine the intervals on which f is increasing and on which f is decreasing.

53. $f(x) = x^3 - 4x^2$

54. $f(x) = |x^4 - 16|$

55–58 ■ Find the average rate of change of the function between the given points.

55. $f(x) = x^2 + 3x; \quad x = 0, x = 2$

56. $f(x) = \dfrac{1}{x - 2}; \quad x = 4, x = 8$

57. $f(x) = \dfrac{1}{x}; \quad x = 3, x = 3 + h$

58. $f(x) = (x + 1)^2; \quad x = a, x = a + h$

59. The population of a planned seaside community in Florida is given by the function $P(t) = 3000 + 200t + 0.1t^2$, where t represents the number of years since the community was incorporated in 1985.
(a) Find $P(10)$ and $P(20)$. What do these values represent?
(b) Find the average rate of change of P between $t = 10$ and $t = 20$. What does this number represent?

60. Ella is saving for her retirement by making regular deposits into a 401(k) plan. As her salary rises, she finds that she can deposit increasing amounts each year. Between 1995 and 2008, the annual amount (in dollars) that she deposited was given by the function $D(t) = 3500 + 15t^2$, where t represents the year of the deposit measured from the start of the plan (so 1995 corresponds to $t = 0$ and 1996 corresponds to $t = 1$, and so on).
(a) Find $D(0)$ and $D(15)$. What do these values represent?
(b) Assuming that her deposits continue to be modeled by the function D, in what year will she deposit \$17,000?
(c) Find the average rate of change of D between $t = 0$ and $t = 15$. What does this number represent?

61–62 ■ A function f is given. (a) Find the average rate of change of f between $x = 0$ and $x = 2$, and the average rate of change of f between $x = 15$ and $x = 50$. (b) Were the two average rates of change that you found in part (a) the same? Explain why or why not.

61. $f(x) = \frac{1}{2}x - 6$

62. $f(x) = 8 - 3x$

63. Suppose the graph of f is given. Describe how the graphs of the following functions can be obtained from the graph of f.
(a) $y = f(x) + 8$
(b) $y = f(x + 8)$
(c) $y = 1 + 2f(x)$
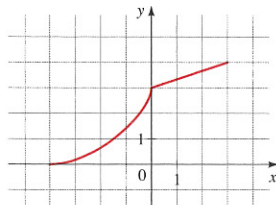(d) $y = f(x - 2) - 2$
(e) $y = f(-x)$
(f) $y = -f(-x)$
(g) $y = -f(x)$
(h) $y = f^{-1}(x)$

64. The graph of f is given. Draw the graphs of the following functions.
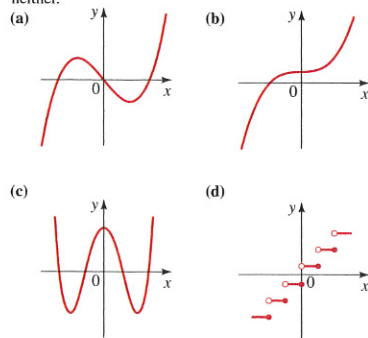
(a) $y = f(x - 2)$ (b) $y = -f(x)$
(c) $y = 3 - f(x)$ (d) $y = \frac{1}{2}f(x) - 1$
(e) $y = f^{-1}(x)$ (f) $y = f(-x)$

65. Determine whether f is even, odd, or neither.

(a) $f(x) = 2x^5 - 3x^2 + 2$ (b) $f(x) = x^3 - x^7$
(c) $f(x) = \dfrac{1 - x^2}{1 + x^2}$ (d) $f(x) = \dfrac{1}{x + 2}$

66. Determine whether the function in the figure is even, odd, or neither.

(a)   (b)

(c)   (d)

67. Find the minimum value of the function $g(x) = 2x^2 + 4x - 5$.

68. Find the maximum value of the function $f(x) = 1 - x - x^2$.

69. A stone is thrown upward from the top of a building. Its height (in feet) above the ground after t seconds is given by

$$h(t) = -16t^2 + 48t + 32$$

What maximum height does it reach?

70. The profit P (in dollars) generated by selling x units of a certain commodity is given by

$$P(x) = -1500 + 12x - 0.0004x^2$$

What is the maximum profit, and how many units must be sold to generate it?

71–72 ■ Find the local maximum and minimum values of the function and the values of x at which they occur. State each answer correct to two decimal places.

71. $f(x) = 3.3 + 1.6x - 2.5x^3$ **72.** $f(x) = x^{2/3}(6 - x)^{1/3}$

73–74 ■ Two functions, f and g, are given. Draw graphs of f, g, and $f + g$ on the same graphing calculator screen to illustrate the concept of graphical addition.

73. $f(x) = x + 2, \quad g(x) = x^2$

74. $f(x) = x^2 + 1, \quad g(x) = 3 - x^2$

75. If $f(x) = x^2 - 3x + 2$ and $g(x) = 4 - 3x$, find the following functions.

(a) $f + g$ (b) $f - g$ (c) fg
(d) f/g (e) $f \circ g$ (f) $g \circ f$

76. If $f(x) = 1 + x^2$ and $g(x) = \sqrt{x - 1}$, find the following.

(a) $f \circ g$ (b) $g \circ f$ (c) $(f \circ g)(2)$
(d) $(f \circ f)(2)$ (e) $f \circ g \circ f$ (f) $g \circ f \circ g$

77–78 ■ Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

77. $f(x) = 3x - 1, \quad g(x) = 2x - x^2$

78. $f(x) = \sqrt{x}, \quad g(x) = \dfrac{2}{x - 4}$

79. Find $f \circ g \circ h$, where $f(x) = \sqrt{1 - x}$, $g(x) = 1 - x^2$, and $h(x) = 1 + \sqrt{x}$.

80. If $T(x) = \dfrac{1}{\sqrt{1 + \sqrt{x}}}$, find functions f, g, and h such that

$$f \circ g \circ h = T.$$

81–86 ■ Determine whether the function is one-to-one.

81. $f(x) = 3 + x^3$

82. $g(x) = 2 - 2x + x^2$

83. $h(x) = \dfrac{1}{x^4}$

84. $r(x) = 2 + \sqrt{x + 3}$

85. $p(x) = 3.3 + 1.6x - 2.5x^3$

86. $q(x) = 3.3 + 1.6x + 2.5x^3$

87–90 ■ Find the inverse of the function.

87. $f(x) = 3x - 2$ **88.** $f(x) = \dfrac{2x + 1}{3}$

89. $f(x) = (x + 1)^3$ **90.** $f(x) = 1 + \sqrt[5]{x - 2}$

91. (a) Sketch the graph of the function

$$f(x) = x^2 - 4 \qquad x \geq 0$$

(b) Use part (a) to sketch the graph of f^{-1}.
(c) Find an equation for f^{-1}.

92. (a) Show that the function $f(x) = 1 + \sqrt[4]{x}$ is one-to-one.
(b) Sketch the graph of f.
(c) Use part (b) to sketch the graph of f^{-1}.
(d) Find an equation for f^{-1}.