

$\sigma_Z^2 = 0.25\sigma_X^2 + 0.25\sigma_Y^2 = 0.5 \times 94,236,826.64 = 47,118,413.32$, so the standard deviation is

$\sigma_Z = \sqrt{0.5\sigma_X^2} \doteq \6864.29 . (b) For 4 men, the expected value of the average income is

$\mu_Z = 0.25\mu_{X_1} + 0.25\mu_{X_2} + 0.25\mu_{X_3} + 0.25\mu_{X_4} = \303.35 ; the same as it was for one policy and two policies. The variance of the average income is

$\sigma_Z^2 = 0.0625\sigma_{X_1}^2 + 0.0625\sigma_{X_2}^2 + 0.0625\sigma_{X_3}^2 + 0.0625\sigma_{X_4}^2 = 0.25\sigma_{X_1}^2 = 23,559,206.66$, so the

standard deviation is $\sigma_Z = \sqrt{0.25\sigma_X^2} \doteq \4853.78 (smaller by a factor of $1/\sqrt{2}$).

7.59 The distribution of the difference $X - Y$ is $N(0, \sqrt{0.3^2 + 0.3^2}) \approx N(0, 0.4243)$ so

$P(|X - Y| \geq 0.8) = P(|Z| \geq 1.8856) = 0.0593$ (Table A gives 0.0588).

7.60 (a) The mean profit is $\mu_X = 1 \times 0.1 + 1.5 \times 0.2 + 2 \times 0.4 + 4 \times 0.2 + 10 \times 0.1 = \3 million. The

variance is $\sigma_X^2 = (1-3)^2 \times 0.1 + (1.5-3)^2 \times 0.2 + (2-3)^2 \times 0.4 + (4-3)^2 \times 0.2 + (10-3)^2 \times 0.1 = 6.35$,

so the standard deviation is $\sigma_X = \sqrt{6.35} \doteq \2.5199 million. (b) The mean and standard deviation

of Y are $\mu_Y = 0.9\mu_X - 0.2 = 0.9 \times \$3 - 0.2 = \$2.5$ million and

$\sigma_Y = \sqrt{0.9^2 \sigma_X^2} = \sqrt{0.9^2 \times 6.35} \doteq \2.2679 million.

7.61 (a) The mean of the difference $Y - X$ is $\mu_{Y-X} = \mu_Y - \mu_X = 2.001 - 2.000 = 0.001$ g. The

variance of the difference is $\sigma_{Y-X}^2 = \sigma_Y^2 + \sigma_X^2 = 0.002^2 + 0.001^2 = 0.000005$ so $\sigma_{Y-X} =$

0.002236g. (b) The expected value of the average is $\mu_Z = \frac{1}{2}\mu_X + \frac{1}{2}\mu_Y = 2.0005$ g. The variance

of the average $\sigma_Z^2 = \frac{1}{4}\sigma_X^2 + \frac{1}{4}\sigma_Y^2 = 0.00000125$, so the standard deviation is $\sigma_Z \doteq 0.001118$ g.

The average Z is slightly more variable than the reading Y , since $\sigma_Z > \sigma_Y$.

7.62 (a) To do one repetition, start at any point in Table B and begin reading digits. As in Example 6.6, let the digits 0, 1, 2, 3, 4 = girl and 5, 6, 7, 8, 9 = boy, and read a string of digits until a "0 to 4" (girl) appears or until four consecutive "5 to 9"s (boys) have appeared, whichever comes first. Then let the observation of X = number of children for this repetition = the number of digits in the string you have read. Repeat this procedure 25 times. (b) The possible outcomes and their corresponding values of X = number of children are shown in the table below.

Outcome		
G	(first child is a girl)	$X=1$
BG	(second child is a girl)	$X=2$
BBG	(third child is a girl)	$X=3$
BBBG, BBBB	(fourth child is a girl or four boys)	$X=4$

Since births are independent and B and G are equally likely to occur on any one birth, we can use our basic probability rules to calculate

$$P(X = 1) = 1/2$$

$$P(X = 2) = (1/2) \times (1/2) = 1/4$$

$$P(X = 3) = (1/2) \times (1/2) \times (1/2) = 1/8$$

$$P(X = 4) = (1/2) \times (1/2) \times (1/2) \times (1/2) + (1/2) \times (1/2) \times (1/2) \times (1/2) = 1/16 + 1/16 = 1/8$$

Thus, the probability distribution of X is

x	1	2	3	4
$P(X = x)$	1/2	1/4	1/8	1/8

(c) The mean number of children with this plan is $\mu_X = 1 \times 1/2 + 2 \times 1/4 + 3 \times 1/8 + 4 \times 1/8 = 1.875$ children.

7.63 (a) A single random digit simulates each toss, with (say) odd = heads and even = tails. The first round is two digits, with two odds a win; if you don't win, look at two more digits, again with two odds a win. Using a calculator, you could use `randInt(0, 1, 2)` which provides 2 digits either a 0 (tail) or 1(head). (b) Using a calculator, in 50 plays (remember, unless you win, a "play" consists of "4 tosses of the coin" or 2 simulations of obtaining 2 random numbers) I obtained 25 wins for an estimate of \$0. (c) The monetary outcome X can be \$1 or -\$1. To win a dollar, you can win on the first round by getting 2 heads or by winning on the second round by not getting 2 heads on the first round, and then getting two heads on the second round. So the

probability of winning is $\frac{1}{4} + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \frac{7}{16}$. So, the expected value is

$$(\$1)\left(\frac{7}{16}\right) + (-\$1)\left(\frac{9}{16}\right) = -\frac{2}{16} = \$0.125.$$

7.64 (a) The value of d_1 is $2 \times 0.002 = 0.004$ and the value of d_2 is $2 \times 0.001 = 0.002$. (b) The standard deviation of the total length $X + Y + Z$ is $\sigma_{X+Y+Z} = \sqrt{0.001^2 + 0.002^2 + 0.001^2} \doteq 0.0024$, so $d \doteq 0.005$ —considerably less than $d_1 + 2d_2 = 0.008$. The engineer was incorrect.