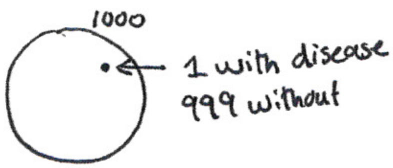
Name: Mr. Young - Rubric Date: 11/7/2013 Prd: 1/2/3

HW 28!! BAYESIAN PROBABILITY QUIZ REVIEW!

Below are some Bayesian probability problems and discrete probability problems. Choose 4 problems from (1-8) and either 9 or 10. Of course you might want to do them all.

1.) A MAN TESTS POSITIVE FOR A GENETIC DISEASE KNOWN AS SICKLE CELL ANEMIA WHICH EFFECTS BLOOD CELLS AND OXYGEN USE. THE TEST FOR THE DISEASE IS 94.2% ACCURATE. IF THE PREVALENCE OF THE DISEASE IN SOCIETY IS 1 IN EVERY 1,000, WHAT IS THE PROBABILITY THAT THE PERSON WHO TESTS POSITIVE ACTUALLY HAS THE DISEASE?



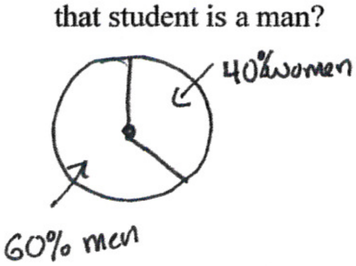
disease, positive

$$P(D|+) = \frac{P(+|D)(P(D))}{P(+)}$$

$$= \frac{(0.942)(1)}{\underbrace{(0.942)(1)}_{\text{should have tested Pos.}} + \underbrace{(0.058)(999)}_{\text{false positive}}} = \boxed{0.016}$$

1.6% Chance that you have the disease given that you tested positive.

2.) A class consists of 60% men and 40% women. Of the men, 25% are blond, while 45% of the women are blond. If a student is chosen at random and is found to be blond, what is the probability that student is a man?



man, blond

$$P(M|B) = \frac{P(B|M)\cdot P(M)}{P(B)}$$

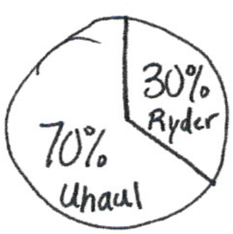
$$= \frac{(0.25)(0.60)}{(0.25)(0.60) + (0.45)(0.40)}$$

45% Chance that the randomly Chosen blond student is a man.

$$\boxed{= 0.45}$$

3) A hauling truck was involved in a hit and run accident at night. Two truck companies, the Uhaul and Ryder, operate in the city. It is known that 70% of the trucks in the city are Uhaul and 30% are Ryder. A witness identified the truck as Ryder. The court tested the reliability of the witness under the circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two truck types 65% of the time and failed 35% of the time.

What is the probability that the truck involved in the accident was Ryder given that the witness identified the truck as being Ryder?



Ryder, Identified as Ryder

$$P(R|I) = \frac{P(I|R)\cdot P(R)}{P(I)}$$

$$= \frac{(0.65)(0.30)}{(0.65)(0.30) + (0.35)(0.70)}$$

$$= \boxed{0.44}$$

44% Chance that the truck involved in the accident was Ryder given that they identified it as Ryder.

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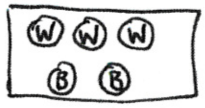
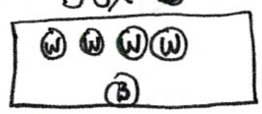
4.) Suppose that Lydia selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains three white balls and two blue balls. The second box contains four white balls and one blue ball. What is the probability that Lydia picked a ball from the first box if she selected a blue ball?

Box A



2/5 Blue

Box B



1/5 Blue

$$P(\underset{\text{Box A}}{A} \mid \underset{\text{Blue}}{Blue}) = \frac{P(Blue \mid A) \cdot P(A)}{P(Blue)}$$

0.50 because she picks a box randomly.

$$= \frac{(2/5)\cdot(0.50)}{(2/5)(0.5) + (1/5)(0.5)}$$

$$= \boxed{0.6666}$$

66.66% chance that Lydia picked from Box 1 (A) given that she picked a blue ball.

5.) Suppose that 3% of all baseball players use steroids, that a baseball player who uses steroids tests positive 98% of the time, and that a player who does not use steroids tests positive 5% of the time. What is the probability that a player who tests positive for steroids actually uses steroids?

False Positive

3% steroids

97% don't use

$$P(\underset{\text{actually use steroids}}{S} \mid \underset{\text{test positive}}{+}) = \frac{P(+ \mid S)(P(S))}{P(+)}$$

$$= \frac{(0.98)(0.03)}{(0.98)(0.03) + (0.05)(0.97)}$$

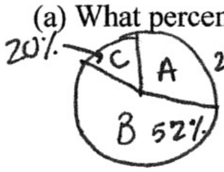
false Positives

$$\boxed{= 0.38}$$

38% chance that the player who tests positive actually uses steroids!

6.) An auto insurance company charges younger drivers a higher premium than it does older drivers because younger drivers as a group tend to have more accidents. The company has 3 age groups: Group A includes those under 25 years old, 28% of all its policyholders. Group B includes those 25-39 years old, 52% of all its policyholders, Group C includes those 40 years old and older. Company records show that in any given one-year period, 23% of its Group A policyholders have an accident. The percentages for groups B and C are 2% and 9%, respectively.

(a) What percent of the company's policyholders are expected to have an accident during the next year?



$$(0.28)(0.23) + (0.52)(0.02) + (0.20)(0.09) =$$

$$\boxed{= 0.0928}$$

(b) Suppose a person has just had an accident. If he/she is one of the policyholders, calculate the probability that he/she is over 40 years old.

$$P(\underset{\text{Group C}}{C} \mid Accident) = \frac{P(Acc. \mid C) \cdot P(C)}{P(Acc.)}$$

$$= \frac{(0.09)(0.20)}{(0.0928)} = \boxed{0.19}$$

19% chance that the person just involved in an accident is over 40 yrs. old.

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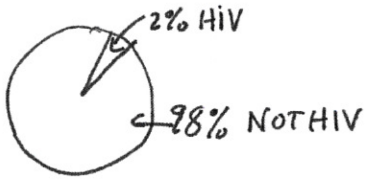
7.) A man was initially thought to have a 2% risk of HIV but ended up with a positive test result (the test accurately classifies about 90% of people with HIV and 95% of people who do not have HIV.) What is the probability that the man who tested positive actually has HIV



$$P(HIV \mid \overset{\text{Positive}}{+}) = \frac{P(+ \mid HIV)\cdot P(HIV)}{P(+)}$$

$$= \frac{(0.90)(0.02)}{(0.90)(0.02) + (0.95)(0.98)}$$

$$= \boxed{26.87\%} \text{ false positives.}$$

26.87% chance that the man who ~~te~~sted positive actually has H.I.V.

Part b.) $P(\text{has HIV} \mid -) = \dfrac{P(- \mid \text{has HIV})\cdot P(HIV)}{P(-)} = \dfrac{(0.1)(0.02)}{(0.1)(0.02) + (0.95)(0.98)} = 0.0021$

0.2% Chance that the man who tested negative actually has HIV.

8.) WRITING: You are a lawyer trying to convict a man being tried for murder for killing their wife. The lawyer on the defense makes the statement that only one in a million men kills their wife so the chances that your client killed their spouse is highly unlikely as well. As the prosecutor, what Bayesian statistic could you bring up (i.e. what conditional probability problem could you present).

$P(H \mid K)$

H: husband committed crime.

K: ~~Killed your spo~~ spouse was killed

The important statistic here is not how many husbands murder their wife (very rare) but on the contrary, Given that someone's wife was killed, what's the probability that the husband committed the crime! $P(H \mid K)$. In other words when someone's wife is murdered how frequently is the culprit the husband.

9.) A die is rolled and a marble is picked out of a bag with 3 red marbles and 2 green marbles. Two events are defined.

A: {a number greater than 2 is rolled}
B: {a green marble is picked}

Find P(A) $= \frac{4}{6} = \boxed{\frac{2}{3}}$

Find P(B) $= \boxed{\frac{2}{5}}$

Find P(!B) $= 1 - \frac{2}{5} = \boxed{\frac{3}{5}}$

Find P(A ∩ B) $= P(A)\cdot P(B) = \frac{2}{3}\cdot\frac{2}{5} = \boxed{\frac{4}{15}}$

Find P(A U B) $= P(A) + P(B) - P(A \cap B) = \frac{2}{3} + \frac{2}{5} - \frac{4}{15} = \frac{10}{15} + \frac{6}{15} - \frac{4}{15} = \frac{12}{15} = \boxed{\frac{4}{5}}$

Find P(A|B)

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{(4/15)}{2/5} = \frac{4}{15}\cdot\frac{5}{2} = \frac{20}{30} = \boxed{\frac{2}{3}}$$

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10.) 3 coins are flipped.

Let A: {At least 2 Tails are observed}
Let B: {At least 1 head is observed}

$P(!A)$

HHH
HHT
HTH
THH

$P(!A \cap B) = \frac{4}{8} = \frac{1}{2}$

a.) List the Sample Space

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

b.) Find P(A)

$P(A) = \frac{4}{8} = \boxed{\frac{1}{2}}$

c.) Find P(B) $P(B) = \boxed{\frac{7}{8}}$

d.) Find $P(!A) = 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$

e.) Find P(A ∩ B)

$P(A \cap B) = \boxed{\frac{3}{8}}$

f.) Find P (A U B) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{7}{8} - \frac{3}{8}$

$= \frac{8}{8} = \boxed{1}$

g) Find P(A|B)

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{(3/8)}{7/8}$ $\quad \frac{3}{\cancel{8}} \cdot \frac{\cancel{8}}{7} = \boxed{\frac{3}{7}}$

h) Find P(B|A) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{(3/8)}{(1/2)} = \frac{3}{8} \cdot \frac{2}{1} = \frac{6}{8} = \boxed{\frac{3}{4}}$

I) Find $P(!A|B) = \frac{P(!A \cap B)}{P(B)} = \frac{1/2}{7/8} = \frac{1}{2} \cdot \frac{8}{7} = \frac{8}{14} = \boxed{\frac{4}{7}}$