2.5 EXERCISES

CONCEPTS

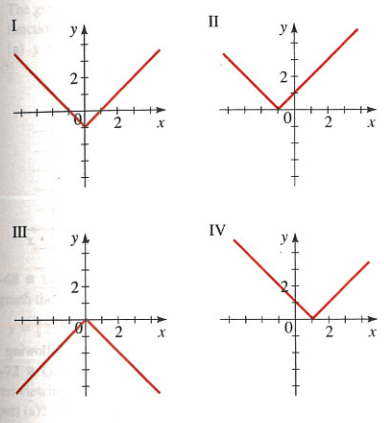
1–2 ■ Fill in the blank with the appropriate direction (left, right, up, or down).

1. **(a)** The graph of $y = f(x) + 3$ is obtained from the graph of $y = f(x)$ by shifting _____ 3 units.
(b) The graph of $y = f(x + 3)$ is obtained from the graph of $y = f(x)$ by shifting _____ 3 units.

2. **(a)** The graph of $y = f(x) - 3$ is obtained from the graph of $y = f(x)$ by shifting _____ 3 units.
(b) The graph of $y = f(x - 3)$ is obtained from the graph of $y = f(x)$ by shifting _____ 3 units.

3. Fill in the blank with the appropriate axis (x-axis or y-axis).
(a) The graph of $y = -f(x)$ is obtained from the graph of $y = f(x)$ by reflecting in the _____.
(b) The graph of $y = f(-x)$ is obtained from the graph of $y = f(x)$ by reflecting in the _____.

4. Match the graph with the function.
(a) $y = |x + 1|$ **(b)** $y = |x - 1|$
(c) $y = |x| - 1$ **(d)** $y = -|x|$



SKILLS

5–14 ■ Suppose the graph of f is given. Describe how the graph of each function can be obtained from the graph of f.

5. **(a)** $y = f(x) - 5$ **(b)** $y = f(x - 5)$
6. **(a)** $y = f(x + 7)$ **(b)** $y = f(x) + 7$
7. **(a)** $y = -f(x)$ **(b)** $y = f(-x)$
8. **(a)** $y = -2f(x)$ **(b)** $y = -\frac{1}{2}f(x)$
9. **(a)** $y = -f(x) + 5$ **(b)** $y = 3f(x) - 5$
10. **(a)** $y = f(x - 4) + \frac{3}{4}$ **(b)** $y = f(x + 4) - \frac{3}{4}$
11. **(a)** $y = 2f(x + 1) - 3$ **(b)** $y = 2f(x - 1) + 3$
12. **(a)** $y = 3 - 2f(x)$ **(b)** $y = 2 - f(-x)$
13. **(a)** $y = f(4x)$ **(b)** $y = f(\frac{1}{4}x)$
14. **(a)** $y = f(2x) - 1$ **(b)** $y = 2f(\frac{1}{2}x)$

15–18 ■ Explain how the graph of g is obtained from the graph of f.

15. **(a)** $f(x) = x^2$, $g(x) = (x + 2)^2$
(b) $f(x) = x^2$, $g(x) = x^2 + 2$

16. **(a)** $f(x) = x^3$, $g(x) = (x - 4)^3$
(b) $f(x) = x^3$, $g(x) = x^3 - 4$

17. **(a)** $f(x) = |x|$, $g(x) = |x + 2| - 2$
(b) $f(x) = |x|$, $g(x) = |x - 2| + 2$

18. **(a)** $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x} + 1$
(b) $f(x) = \sqrt{x}$, $g(x) = \sqrt{-x} + 1$

19. Use the graph of $y = x^2$ in Figure 4 to graph the following.
(a) $g(x) = x^2 + 1$
(b) $g(x) = (x - 1)^2$
(c) $g(x) = -x^2$
(d) $g(x) = (x - 1)^2 + 3$

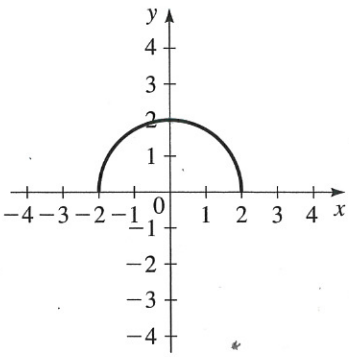
20. Use the graph of $y = \sqrt{x}$ in Figure 5 to graph the following.
(a) $g(x) = \sqrt{x - 2}$
(b) $g(x) = \sqrt{x} + 1$
(c) $g(x) = \sqrt{x + 2} + 2$
(d) $g(x) = -\sqrt{x} + 1$

21–44 ■ Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

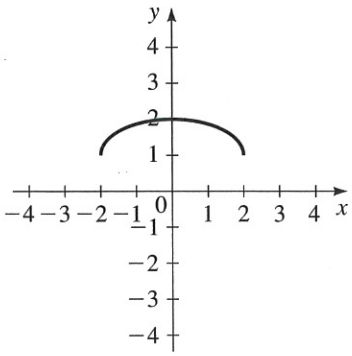
21. $f(x) = x^2 - 1$
22. $f(x) = x^2 + 5$
23. $f(x) = \sqrt{x} + 1$
24. $f(x) = |x| - 1$
25. $f(x) = (x - 5)^2$
26. $f(x) = (x + 1)^2$
27. $f(x) = \sqrt{x + 4}$
28. $f(x) = |x - 3|$
29. $f(x) = -x^3$
30. $f(x) = -|x|$
31. $y = \sqrt[4]{-x}$
32. $y = \sqrt[3]{-x}$
33. $y = \frac{1}{4}x^2$

DRILL QUESTION

Given the graph of $f(x)$ shown, sketch the graph of $\frac{1}{2}f(x) + 1$.



Answer



34. $y = -5\sqrt{x}$

35. $y = 3|x|$

36. $y = \frac{1}{2}|x|$

37. $y = (x - 3)^2 + 5$

38. $y = \sqrt{x + 4} - 3$

39. $y = 3 - \frac{1}{2}(x - 1)^2$

40. $y = 2 - \sqrt{x + 1}$

41. $y = |x + 2| + 2$

42. $y = 2 - |x|$

43. $y = \frac{1}{2}\sqrt{x + 4} - 3$

44. $y = 3 - 2(x - 1)^2$

45–54 ■ A function f is given, and the indicated transformations are applied to its graph (in the given order). Write the equation for the final transformed graph.

45. $f(x) = x^2$; shift upward 3 units

46. $f(x) = x^3$; shift downward 1 unit

47. $f(x) = \sqrt{x}$; shift 2 units to the left

48. $f(x) = \sqrt[3]{x}$; shift 1 unit to the right

49. $f(x) = |x|$; shift 3 units to the right and shift upward 1 unit

50. $f(x) = |x|$; shift 4 units to the left and shift downward 2 units

51. $f(x) = \sqrt[4]{x}$; reflect in the y-axis and shift upward 1 unit

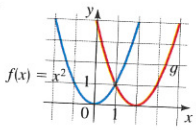
52. $f(x) = x^2$; shift 2 units to the left and reflect in the x-axis

53. $f(x) = x^2$; stretch vertically by a factor of 2, shift downward 2 units, and shift 3 units to the right

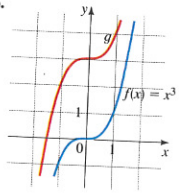
54. $f(x) = |x|$; shrink vertically by a factor of $\frac{1}{2}$, shift to the left 1 unit, and shift upward 3 units

55–60 ■ The graphs of f and g are given. Find a formula for the function g.

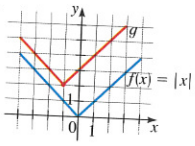
55.



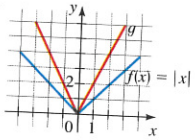
56.



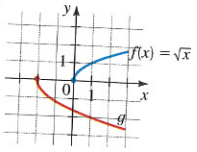
57.



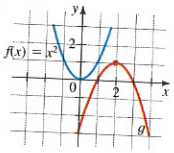
58.



59.

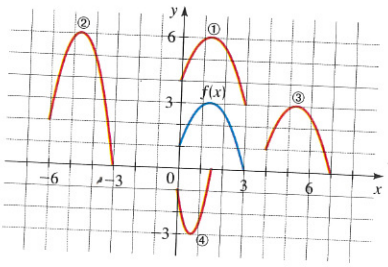


60.

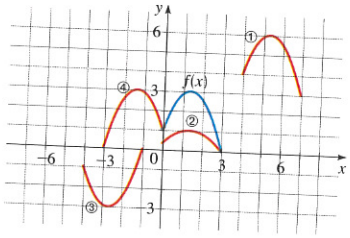


61–62 ■ The graph of $y = f(x)$ is given. Match each equation with its graph.

61. **(a)** $y = f(x - 4)$ **(b)** $y = f(x) + 3$
(c) $y = 2f(x + 6)$ **(d)** $y = -f(2x)$

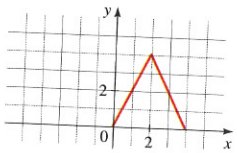


62. **(a)** $y = \frac{1}{3}f(x)$ **(b)** $y = -f(x + 4)$
(c) $y = f(x - 4) + 3$ **(d)** $y = f(-x)$



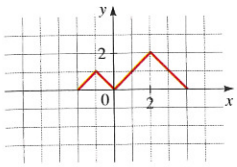
63. The graph of f is given. Sketch the graphs of the following functions.

(a) $y = f(x - 2)$ **(b)** $y = f(x) - 2$
(c) $y = 2f(x)$ **(d)** $y = -f(x) + 3$
(e) $y = f(-x)$ **(f)** $y = \frac{1}{2}f(x - 1)$



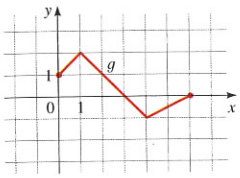
64. The graph of g is given. Sketch the graphs of the following functions.

(a) $y = g(x + 1)$ **(b)** $y = g(-x)$
(c) $y = g(x - 2)$ **(d)** $y = g(x) - 2$
(e) $y = -g(x)$ **(f)** $y = 2g(x)$



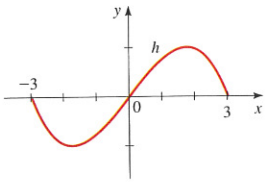
65. The graph of g is given. Use it to graph each of the following functions.

(a) $y = g(2x)$ **(b)** $y = g(\frac{1}{2}x)$



66. The graph of h is given. Use it to graph each of the following functions.

(a) $y = h(3x)$ **(b)** $y = h(\frac{1}{3}x)$



67–68 ■ Use the graph of $f(x) = [\![x]\!]$ described on page 156 to graph the indicated function.

67. $y = [\![2x]\!]$ **68.** $y = [\![\frac{1}{4}x]\!]$

69–72 ■ Graph the functions on the same screen using the given viewing rectangle. How is each graph related to the graph in part (a)?

69. Viewing rectangle $[-8, 8]$ by $[-2, 8]$
(a) $y = \sqrt[4]{x}$ **(b)** $y = \sqrt[4]{x + 5}$
(c) $y = 2\sqrt[4]{x + 5}$ **(d)** $y = 4 + 2\sqrt[4]{x + 5}$

70. Viewing rectangle $[-8, 8]$ by $[-6, 6]$
(a) $y = |x|$ **(b)** $y = -|x|$
(c) $y = -3|x|$ **(d)** $y = -3|x - 5|$

71. Viewing rectangle $[-4, 6]$ by $[-4, 4]$
(a) $y = x^6$ **(b)** $y = \frac{1}{3}x^6$
(c) $y = -\frac{1}{3}x^6$ **(d)** $y = -\frac{1}{3}(x - 4)^6$

72. Viewing rectangle $[-6, 6]$ by $[-4, 4]$
(a) $y = \dfrac{1}{\sqrt{x}}$ **(b)** $y = \dfrac{1}{\sqrt{x + 3}}$
(c) $y = \dfrac{1}{2\sqrt{x + 3}}$ **(d)** $y = \dfrac{1}{2\sqrt{x + 3}} - 3$

73. If $f(x) = \sqrt{2x - x^2}$, graph the following functions in the viewing rectangle $[-5, 5]$ by $[-4, 4]$. How is each graph related to the graph in part (a)?
(a) $y = f(x)$ **(b)** $y = f(2x)$ **(c)** $y = f(\frac{1}{2}x)$

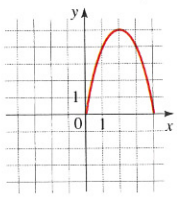
74. If $f(x) = \sqrt{2x - x^2}$, graph the following functions in the viewing rectangle $[-5, 5]$ by $[-4, 4]$. How is each graph related to the graph in part (a)?
(a) $y = f(x)$ **(b)** $y = f(-x)$
(c) $y = -f(-x)$ **(d)** $y = f(-2x)$
(e) $y = f(-\frac{1}{2}x)$

75–82 ■ Determine whether the function f is even, odd, or neither. If f is even or odd, use symmetry to sketch its graph.

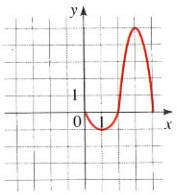
75. $f(x) = x^4$ **76.** $f(x) = x^3$
77. $f(x) = x^2 + x$ **78.** $f(x) = x^4 - 4x^2$
79. $f(x) = x^3 - x$ **80.** $f(x) = 3x^3 + 2x^2 + 1$
81. $f(x) = 1 - \sqrt[3]{x}$ **82.** $f(x) = x + \dfrac{1}{x}$

83–84 ■ The graph of a function defined for $x \geq 0$ is given. Complete the graph for $x < 0$ to make **(a)** an even function and **(b)** an odd function.

83.

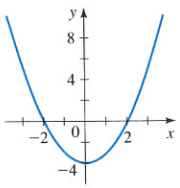


84.

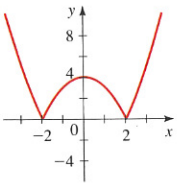


85–86 ■ These exercises show how the graph of $y = |f(x)|$ is obtained from the graph of $y = f(x)$.

85. The graphs of $f(x) = x^2 - 4$ and $g(x) = |x^2 - 4|$ are shown. Explain how the graph of g is obtained from the graph of f.



$f(x) = x^2 - 4$



$g(x) = |x^2 - 4|$