

**ALTERNATE EXAMPLE 7**

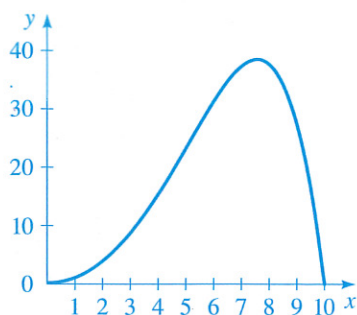
The profit, in millions of dollars, a factory makes by producing  $x$  thousand items is approximated by the function

$$f(x) = -\frac{x^6}{10000} + x^2$$

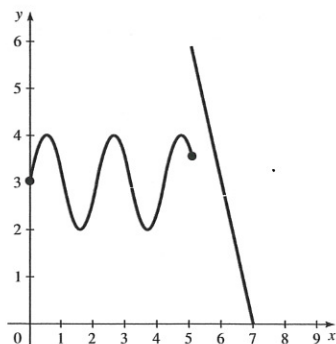
when  $x$  is between 0 and 10,000. Estimate the amount of items that the factory should make to maximize its profit.

**ANSWER**

The graph of  $f$  as a function of  $x$  is shown here. There appears to be a maximum between  $x = 7$  and  $x = 9$ . Using the *maximum* command, we see that the maximum value of  $f$  is 38.49, or \$38,490,000. It occurs when the factory produces 7598 items.

**DRILL QUESTION**

What are the domain and range of this function?

**Answer**

Domain:  $0 \leq x \leq 7$  or  $x \leq 7$

Range:  $0 \leq y \leq 6$

**EXAMPLE 7 | A Model for the Food Price Index**

A model for the food price index (the price of a representative “basket” of foods) between 1990 and 2000 is given by the function

$$I(t) = -0.0113t^3 + 0.0681t^2 + 0.198t + 99.1$$

where  $t$  is measured in years since midyear 1990, so  $0 \leq t \leq 10$ , and  $I(t)$  is scaled so that  $I(3) = 100$ . Estimate the time when food was most expensive during the period 1990–2000.

**SOLUTION** The graph of  $I$  as a function of  $t$  is shown in Figure 12(a). There appears to be a maximum between  $t = 4$  and  $t = 7$ . Using the *maximum* command, as shown in Figure 12(b), we see that the maximum value of  $I$  is about 100.38, and it occurs when  $t \approx 5.15$ , which corresponds to August 1995.

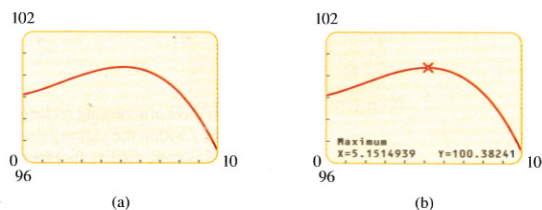
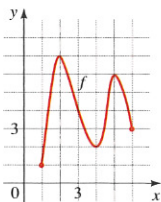


FIGURE 12

■ NOW TRY EXERCISE 53

**2.3 EXERCISES****CONCEPTS**

1–4 ■ These exercises refer to the graph of the function  $f$  shown below.

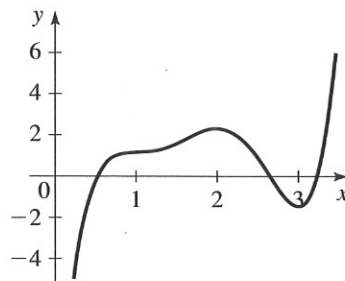


- To find a function value  $f(a)$  from the graph of  $f$ , we find the height of the graph above the  $x$ -axis at  $x =$  \_\_\_\_\_. From the graph of  $f$  we see that  $f(3) =$  \_\_\_\_\_.
- The domain of the function  $f$  is all the \_\_\_\_\_-values of the points on the graph, and the range is all the corresponding \_\_\_\_\_-values. From the graph of  $f$  we see that the domain of  $f$  is the interval \_\_\_\_\_ and the range of  $f$  is the interval \_\_\_\_\_.

- If  $f$  is increasing on an interval, then the  $y$ -values of the points on the graph \_\_\_\_\_ as the  $x$ -values increase. From the graph of  $f$  we see that  $f$  is increasing on the intervals \_\_\_\_\_ and \_\_\_\_\_.
  - If  $f$  is decreasing on an interval, then  $y$ -values of the points on the graph \_\_\_\_\_ as the  $x$ -values increase. From the graph of  $f$  we see that  $f$  is decreasing on the intervals \_\_\_\_\_ and \_\_\_\_\_.
- A function value  $f(a)$  is a local maximum value of  $f$  if  $f(a)$  is the \_\_\_\_\_ value of  $f$  on some interval containing  $a$ . From the graph of  $f$  we see that one local maximum value of  $f$  is \_\_\_\_\_ and that this value occurs when  $x$  is \_\_\_\_\_.
  - The function value  $f(a)$  is a local minimum value of  $f$  if  $f(a)$  is the \_\_\_\_\_ value of  $f$  on some interval containing  $a$ . From the graph of  $f$  we see that one local minimum value of  $f$  is \_\_\_\_\_ and that this value occurs when  $x$  is \_\_\_\_\_.

**EXAMPLE**

A function with two integer turning points and a flat spot:

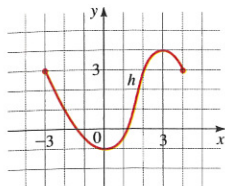


$$\frac{1}{6}(12x^5 - 105x^4 + 340x^3 - 510x^2 + 360x - 90)$$

## SKILLS

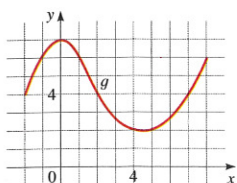
5. The graph of a function  $h$  is given.

- Find  $h(-2)$ ,  $h(0)$ ,  $h(2)$ , and  $h(3)$ .
- Find the domain and range of  $h$ .
- Find the values of  $x$  for which  $h(x) = 3$ .
- Find the values of  $x$  for which  $h(x) \leq 3$ .



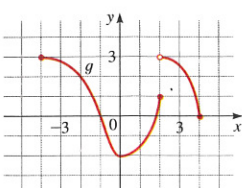
6. The graph of a function  $g$  is given.

- Find  $g(-2)$ ,  $g(0)$ , and  $g(7)$ .
- Find the domain and range of  $g$ .
- Find the values of  $x$  for which  $g(x) = 4$ .
- Find the values of  $x$  for which  $g(x) > 4$ .



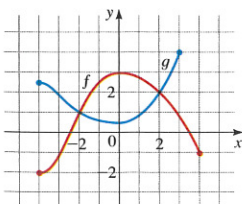
7. The graph of a function  $g$  is given.

- Find  $g(-4)$ ,  $g(-2)$ ,  $g(0)$ ,  $g(2)$ , and  $g(4)$ .
- Find the domain and range of  $g$ .



8. Graphs of the functions  $f$  and  $g$  are given.

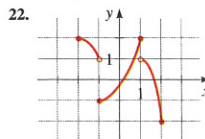
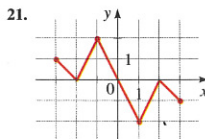
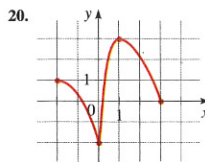
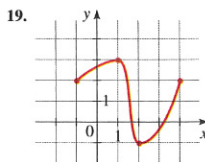
- Which is larger,  $f(0)$  or  $g(0)$ ?
- Which is larger,  $f(-3)$  or  $g(-3)$ ?
- For which values of  $x$  is  $f(x) = g(x)$ ?



9–18 ■ A function  $f$  is given. (a) Use a graphing calculator to draw the graph of  $f$ . (b) Find the domain and range of  $f$  from the graph.

- $f(x) = x - 1$
- $f(x) = 2(x + 1)$
- $f(x) = 4, \quad 1 \leq x \leq 3$
- $f(x) = x^2, \quad -2 \leq x \leq 5$
- $f(x) = 4 - x^2$
- $f(x) = x^2 + 4$
- $f(x) = \sqrt{16 - x^2}$
- $f(x) = -\sqrt{25 - x^2}$
- $f(x) = \sqrt{x - 1}$
- $f(x) = \sqrt{x + 2}$

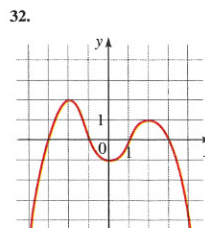
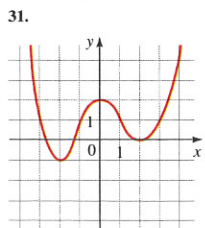
19–22 ■ The graph of a function is given. Determine the intervals on which the function is (a) increasing and (b) decreasing.



23–30 ■ A function  $f$  is given. (a) Use a graphing device to draw the graph of  $f$ . (b) State approximately the intervals on which  $f$  is increasing and on which  $f$  is decreasing.

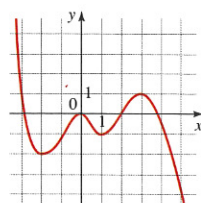
- $f(x) = x^2 - 5x$
- $f(x) = x^3 - 4x$
- $f(x) = 2x^3 - 3x^2 - 12x$
- $f(x) = x^4 - 16x^2$
- $f(x) = x^3 + 2x^2 - x - 2$
- $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$
- $f(x) = x^{2/5}$
- $f(x) = 4 - x^{2/3}$

31–34 ■ The graph of a function is given. (a) Find all the local maximum and minimum values of the function and the value of  $x$  at which each occurs. (b) Find the intervals on which the function is increasing and on which the function is decreasing.

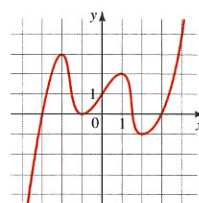




33.



34.



**35–42** ■ A function is given. (a) Find all the local maximum and minimum values of the function and the value of  $x$  at which each occurs. State each answer correct to two decimal places. (b) Find the intervals on which the function is increasing and on which the function is decreasing. State each answer correct to two decimal places.

35.  $f(x) = x^3 - x$

36.  $f(x) = 3 + x + x^2 - x^3$

37.  $g(x) = x^4 - 2x^3 - 11x^2$

38.  $g(x) = x^5 - 8x^3 + 20x$

39.  $U(x) = x\sqrt{6-x}$

40.  $U(x) = x\sqrt{x-x^2}$

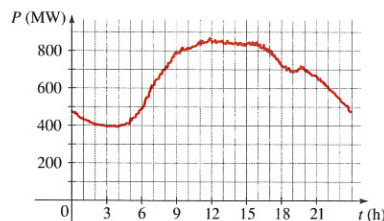
41.  $V(x) = \frac{1-x^2}{x^3}$

42.  $V(x) = \frac{1}{x^2 + x + 1}$

## APPLICATIONS

**43. Power Consumption** The figure shows the power consumption in San Francisco for September 19, 1996 ( $P$  is measured in megawatts;  $t$  is measured in hours starting at midnight).

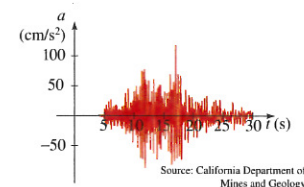
- (a) What was the power consumption at 6:00 A.M.  
At 6:00 P.M.?  
(b) When was the power consumption the lowest?  
(c) When was the power consumption the highest?



Source: Pacific Gas &amp; Electric

**44. Earthquake** The graph shows the vertical acceleration of the ground from the 1994 Northridge earthquake in Los Angeles, as measured by a seismograph. (Here  $t$  represents the time in seconds.)

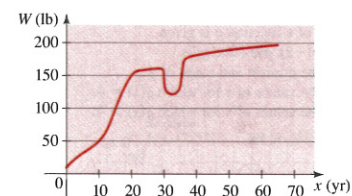
- (a) At what time  $t$  did the earthquake first make noticeable movements of the earth?  
(b) At what time  $t$  did the earthquake seem to end?  
(c) At what time  $t$  was the maximum intensity of the earthquake reached?



Source: California Department of Mines and Geology

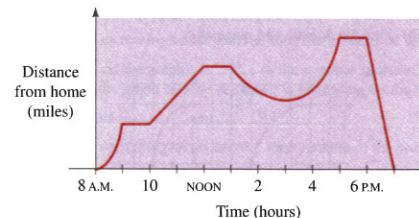
**45. Weight Function** The graph gives the weight  $W$  of a person at age  $x$ .

- (a) Determine the intervals on which the function  $W$  is increasing and those on which it is decreasing.  
(b) What do you think happened when this person was 30 years old?



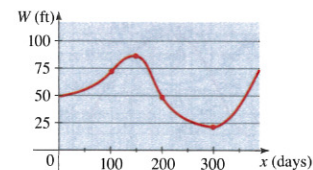
**46. Distance Function** The graph gives a sales representative's distance from his home as a function of time on a certain day.

- (a) Determine the time intervals on which his distance from home was increasing and those on which it was decreasing.  
(b) Describe in words what the graph indicates about his travels on this day.

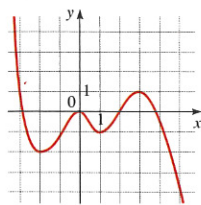


**47. Changing Water Levels** The graph shows the depth of water  $W$  in a reservoir over a one-year period as a function of the number of days  $x$  since the beginning of the year.

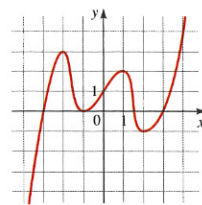
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33.



34.



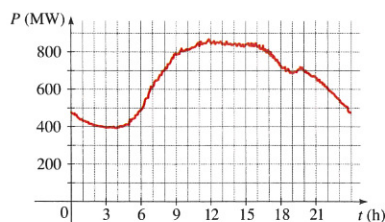
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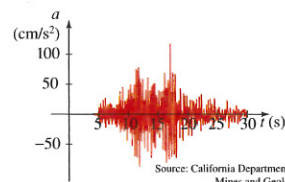
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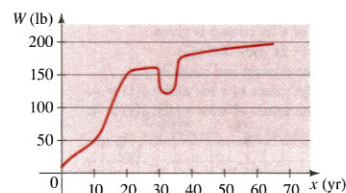
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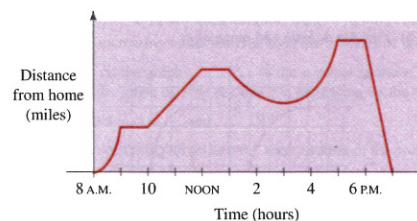
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 A local minimum?

