SKILLS

5–10 ■ The graph of a function f is given. Determine whether f is one-to-one.

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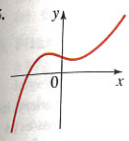

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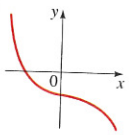

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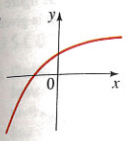

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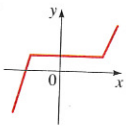

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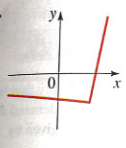

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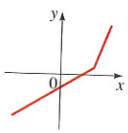


11–20 ■ Determine whether the function is one-to-one.

11. $f(x) = -2x + 4$

12. $f(x) = 3x - 2$

13. $g(x) = \sqrt{x}$

14. $g(x) = |x|$

15. $h(x) = x^2 - 2x$

16. $h(x) = x^3 + 8$

17. $f(x) = x^4 + 5$

18. $f(x) = x^4 + 5, \quad 0 \le x \le 2$

19. $f(x) = \dfrac{1}{x^2}$

20. $f(x) = \dfrac{1}{x}$

21–22 ■ Assume that f is a one-to-one function.

21. (a) If $f(2) = 7$, find $f^{-1}(7)$.
(b) If $f^{-1}(3) = -1$, find $f(-1)$.

22. (a) If $f(5) = 18$, find $f^{-1}(18)$.
(b) If $f^{-1}(4) = 2$, find $f(2)$.

23. If $f(x) = 5 - 2x$, find $f^{-1}(3)$.

24. If $g(x) = x^2 + 4x$ with $x \ge -2$, find $g^{-1}(5)$.

25–36 ■ Use the Inverse Function Property to show that f and g are inverses of each other.

25. $f(x) = x - 6; \quad g(x) = x + 6$

26. $f(x) = 3x; \quad g(x) = \dfrac{x}{3}$

27. $f(x) = 2x - 5; \quad g(x) = \dfrac{x + 5}{2}$

28. $f(x) = \dfrac{3 - x}{4}; \quad g(x) = 3 - 4x$

29. $f(x) = \dfrac{1}{x}; \quad g(x) = \dfrac{1}{x}$

30. $f(x) = x^5; \quad g(x) = \sqrt[5]{x}$

31. $f(x) = x^2 - 4, \quad x \ge 0;$
$g(x) = \sqrt{x + 4}, \quad x \ge -4$

32. $f(x) = x^3 + 1; \quad g(x) = (x - 1)^{1/3}$

33. $f(x) = \dfrac{1}{x - 1}, \quad x \ne 1; \quad g(x) = \dfrac{1}{x} + 1, \quad x \ne 0$

34. $f(x) = \sqrt{4 - x^2}, \quad 0 \le x \le 2;$
$g(x) = \sqrt{4 - x^2}, \quad 0 \le x \le 2$

35. $f(x) = \dfrac{x + 2}{x - 2}; \quad g(x) = \dfrac{2x + 2}{x - 1}$

36. $f(x) = \dfrac{x - 5}{3x + 4}; \quad g(x) = \dfrac{5 + 4x}{1 - 3x}$

37–60 ■ Find the inverse function of f.

37. $f(x) = 2x + 1$

38. $f(x) = 6 - x$

39. $f(x) = 4x + 7$

40. $f(x) = 3 - 5x$

41. $f(x) = 5 - 4x^3$

42. $f(x) = \dfrac{1}{x^2}, \quad x > 0$

43. $f(x) = \dfrac{1}{x + 2}$

44. $f(x) = \dfrac{x - 2}{x + 2}$

45. $f(x) = \dfrac{x}{x + 4}$

46. $f(x) = \dfrac{3x}{x - 2}$

47. $f(x) = \dfrac{2x + 5}{x - 7}$

48. $f(x) = \dfrac{4x - 2}{3x + 1}$

49. $f(x) = \dfrac{1 + 3x}{5 - 2x}$

50. $f(x) = \dfrac{2x - 1}{x - 3}$

51. $f(x) = \sqrt{2 + 5x}$

52. $f(x) = x^2 + x, \quad x \ge -\frac{1}{2}$

53. $f(x) = 4 - x^2, \quad x \ge 0$

54. $f(x) = \sqrt{2x - 1}$

55. $f(x) = 4 + \sqrt[3]{x}$

56. $f(x) = (2 - x^3)^5$

57. $f(x) = 1 + \sqrt{1 + x}$

58. $f(x) = \sqrt{9 - x^2}, \quad 0 \le x \le 3$

59. $f(x) = x^4, \quad x \ge 0$

60. $f(x) = 1 - x^3$

61–64 ■ A function f is given. **(a)** Sketch the graph of f. **(b)** Use the graph of f to sketch the graph of f^{-1}. **(c)** Find f^{-1}.

61. $f(x) = 3x - 6$

62. $f(x) = 16 - x^2, \quad x \ge 0$

63. $f(x) = \sqrt{x + 1}$

64. $f(x) = x^3 - 1$

65–70 ■ Draw the graph of f and use it to determine whether the function is one-to-one.

65. $f(x) = x^3 - x$

66. $f(x) = x^3 + x$

67. $f(x) = \dfrac{x + 12}{x - 6}$

68. $f(x) = \sqrt{x^3 - 4x + 1}$

69. $f(x) = |x| - |x - 6|$

70. $f(x) = x \cdot |x|$

71–74 ■ A one-to-one function is given. **(a)** Find the inverse of the function. **(b)** Graph both the function and its inverse on the same screen to verify that the graphs are reflections of each other in the line $y = x$.

71. $f(x) = 2 + x$

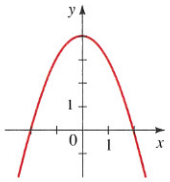
72. $f(x) = 2 - \frac{1}{2}x$

73. $g(x) = \sqrt{x + 3}$

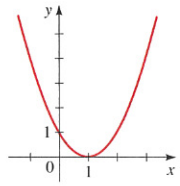
74. $g(x) = x^2 + 1, \quad x \geq 0$

75–78 ■ The given function is not one-to-one. Restrict its domain so that the resulting function *is* one-to-one. Find the inverse of the function with the restricted domain. (There is more than one correct answer.)

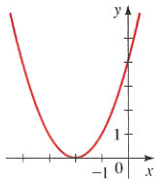
75. $f(x) = 4 - x^2$



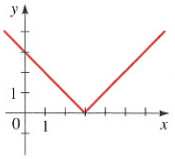
76. $g(x) = (x - 1)^2$



77. $h(x) = (x + 2)^2$

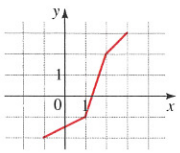


78. $k(x) = |x - 3|$



79–80 ■ Use the graph of f to sketch the graph of f^{-1}.

79.



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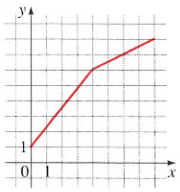

APPLICATIONS

81. Fee for Service For his services, a private investigator requires a $500 retention fee plus $80 per hour. Let x represent the number of hours the investigator spends working on a case.

(a) Find a function f that models the investigator's fee as a function of x.

(b) Find f^{-1}. What does f^{-1} represent?

(c) Find $f^{-1}(1220)$. What does your answer represent?

82. Toricelli's Law A tank holds 100 gallons of water, which drains from a leak at the bottom, causing the tank to empty in 40 minutes. Toricelli's Law gives the volume of water remaining in the tank after t minutes as

$$V(t) = 100\left(1 - \frac{t}{40}\right)^2$$

(a) Find V^{-1}. What does V^{-1} represent?

(b) Find $V^{-1}(15)$. What does your answer represent?

83. Blood Flow As blood moves through a vein or artery, its velocity v is greatest along the central axis and decreases as the distance r from the central axis increases (see the figure below). For an artery with radius 0.5 cm, v (in cm/s) is given as a function of r (in cm) by

$$v(r) = 18{,}500(0.25 - r^2)$$

(a) Find v^{-1}. What does v^{-1} represent?

(b) Find $v^{-1}(30)$. What does your answer represent?

84. Demand Function The amount of a commodity that is sold is called the *demand* for the commodity. The demand D for a certain commodity is a function of the price given by

$$D(p) = -3p + 150$$

(a) Find D^{-1}. What does D^{-1} represent?

(b) Find $D^{-1}(30)$. What does your answer represent?

85. Temperature Scales The relationship between the Fahrenheit (F) and Celsius (C) scales is given by

$$F(C) = \tfrac{9}{5}C + 32$$

(a) Find F^{-1}. What does F^{-1} represent?

(b) Find $F^{-1}(86)$. What does your answer represent?

86. Exchange Rates The relative value of currencies fluctuates every day. When this problem was written, one Canadian dollar was worth 1.0573 U.S. dollar.

(a) Find a function f that gives the U.S. dollar value $f(x)$ of x Canadian dollars.

(b) Find f^{-1}. What does f^{-1} represent?

(c) How much Canadian money would $12,250 in U.S. currency be worth?

87. Income Tax In a certain country, the tax on incomes less than or equal to €20,000 is 10%. For incomes that are more than €20,000, the tax is €2000 plus 20% of the amount over €20,000.

(a) Find a function f that gives the income tax on an income x. Express f as a piecewise defined function.

(b) Find f^{-1}. What does f^{-1} represent?

(c) How much income would require paying a tax of €10,000?

88. Multiple Discounts A car dealership advertises a 15% discount on all its new cars. In addition, the manufacturer offers a $1000 rebate on the purchase of a new car. Let x represent the sticker price of the car.

(a) Suppose only the 15% discount applies. Find a function f that models the purchase price of the car as a function of the sticker price x.