

SECTION II

1. (a) A complete answer compares shape, center, and spread.
Shape. The control group distribution is somewhat bell-shaped and symmetric, while the treatment group distribution is somewhat skewed right.
Center. The center of the control group distribution is around 20, which is greater than the center of the treatment group distribution, which is somewhere around 10 to 12.
Spread. The spread of the control group distribution, 5 to 34, is less than the spread of the treatment group distribution, which is 2 to 41.
- (b) For computers in the control group (no spam software), the number of spam e-mails received varies an “average” amount of 8.1 from the mean number of spam e-mails received in the control group.
- (c) Since the 95% confidence interval for the difference does not contain zero, the researcher can conclude the observed difference in mean numbers of spam e-mails received between the control group and the treatment group that received spam software is significant.
- (d) It may well be that the four groups—administrators, staff, faculty, and students—are each exposed to different kinds of spam e-mail risks, and possibly the software will be more or less of a help to each group. In that case, the researcher should in effect run four separate experiments on the homogeneous groups, called blocks. Conclusions will be more specific.

Scoring

Part (a) is essentially correct for correctly comparing shape, center, and spread.
 Part (a) is partially correct for correctly comparing two of the three features.

Part (b) is essentially correct if standard deviation is explained correctly in context of this problem. Part (b) is partially correct if there is a correct explanation of SD but no reference to context.

Part (c) is essentially correct for noting that zero is not in the interval so the observed difference is significant, and stating this in context of the problem. Part (c) is partially correct for noting that zero is not in the interval so the observed difference is significant, but failing to put this conclusion in context of the problem.

Part (d) is essentially correct if the purpose of blocking is correctly explained in context of this problem. Part (d) is partially correct if the general purpose of blocking is correctly explained but not in context of this problem.

Count partially correct answers as one-half an essentially correct answer.

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| 4 Complete Answer | Four essentially correct answers. |
| 3 Substantial Answer | Three essentially correct answers. |
| 2 Developing Answer | Two essentially correct answers. |
| 1 Minimal Answer | One essentially correct answer. |

Use a holistic approach to decide a score totaling between two numbers.

2. (a) Listing the eight possibilities: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} clearly shows that the only possibilities for the absolute value of the difference are 1 with probability $6/8 = .75$ and 3 with probability $2/8 = .25$. The table is:

Absolute difference	Probability
1	.75
3	.25

(b) $E = \sum xP(x) = 1(.75) + 3(.25) = 1.5$

(c) Since the only possible scores for each game are 1 and 3, the only way to have a total score of 3 in three games is to score 1 in each game. The probability of this is $(.75)^3 = .421875$ [or $\left(\frac{3}{4}\right)^3 = \frac{27}{64}$].

(d) The more times the game is played, the closer the average score will be to the expected value of 1.5. The player does not want to average close to 1.5, so should prefer playing 10 times rather than 15 times.

Scoring

Part (a) is essentially correct for the correct probability distribution table. Part (a) is partially correct for one minor error.

Part (b) is essentially correct for the correct calculation of expected value based on the answer given in Part (a). Part (b) is partially correct for the correct formula for expected value but an incorrect calculation based on the answer given in Part (a).

Part (c) is essentially correct for the correct probability calculation with some indication of where the answer is coming from. Part (c) is partially correct for the correct probability with no work shown.

Part (d) is essentially correct for choosing 10 and giving a clear explanation. Part (d) is partially correct for choosing 10 and giving a weak explanation. Part (d) is incorrect for choosing 10 with no explanation or with an incorrect explanation.

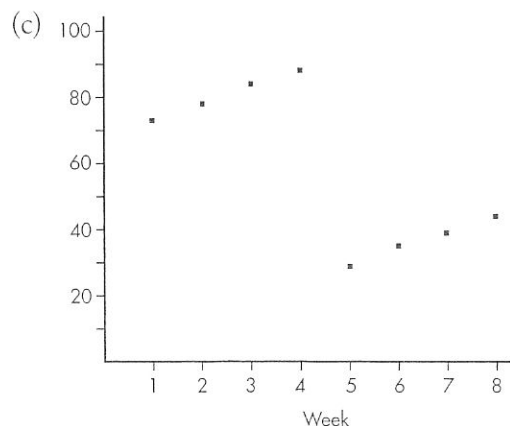
Count partially correct answers as one-half an essentially correct answer.

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|-----------------------------|------------------------------------|
| 4 Complete Answer | Four essentially correct answers. |
| 3 Substantial Answer | Three essentially correct answers. |
| 2 Developing Answer | Two essentially correct answers. |
| 1 Minimal Answer | One essentially correct answer. |

Use a holistic approach to decide a score totaling between two numbers.

3. (a) The slope of -7.19 says that the attendance *dropped* an average of 7.19 students per week. No, this does not seem to adequately explain the data, because the attendance *increased* every week during the first 4 weeks, and again *increased* every week during the final 4 weeks.

(b) Modeling the first 4 weeks (computer games allowed) gives $Attendance = 68 + 5.1(Week)$ with $r = .997$, and modeling the final 4 weeks (no computer games) gives $Attendance = 4.9 + 4.9(Week)$ with $r = .997$. These models give an average increase in attendance each week of 5.1 and 4.9, respectively, as well as much higher correlations.



The scatterplot clearly shows the linearity in the first 4 weeks and in the final 4 weeks, and also the nonlinearity of the full set of data.

Scoring

Part (a) is essentially correct if the slope is correctly interpreted in context and a reasonable explanation is given as to why this slope does not explain the data. Part (a) is partially correct for one of these two components correct.

Part (b) is essentially correct for two separate regression models, correct interpretations of the slopes, and noting the increased correlation. Part (b) is partially correct if one of these components is missing.

Part (c) is essentially correct if the scatterplot is correctly drawn and the strong linearity in the first four weeks and separately in the final four weeks is noted. Part (c) is partially correct for a correctly drawn scatterplot but no observations on linearity.

4 Complete Answer	All three parts essentially correct.
3 Substantial Answer	Two parts essentially correct and one part partially correct.
2 Developing Answer	Two parts essentially correct OR one part essentially correct and one or two parts partially correct OR all three parts partially correct.
1 Minimal Answer	One part essentially correct OR two parts partially correct.

4. This is a paired data test, not a two-sample test, with four parts to a complete solution.

Part 1: Must state a correct pair of hypotheses.

Either $H_0 : \mu_d = 0$ and $H_a : \mu_d < 0$ where μ_d is the mean difference between the investigator and facility estimates; or

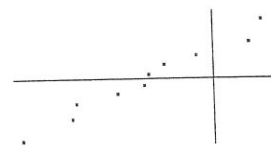
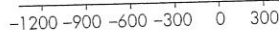
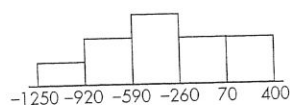
$H_0 : \mu_1 - \mu_2 = 0$ and $H_a : \mu_1 - \mu_2 < 0$ where μ_1 is the mean estimate of the investigator and μ_2 is the mean estimate of the facility.

Part 2: Must name the test and check the conditions.

This is a paired t -test, that is, a single sample hypothesis test on the set of differences.

Conditions:

It is reasonable to assume that the 10 data pairs are independent of each other. Normality of the population distribution of differences should be checked graphically on the sample data using a histogram, or a boxplot, or a normal probability plot:



Part 3: Must find the test statistic t and the P -value.

A calculator quickly gives $t = -2.56$ and $P = .015$.

Or, with $\bar{x}_d = -400.5$ and $s_d = 494.0$ we have $t = \frac{-400.5 - 0}{\frac{494.0}{\sqrt{10}}} = -2.56$

and with $df = 9$, $P = .015$.

Part 4: Linking to the P -value, give a correct conclusion in context.

With this small a P -value (.015), there is evidence to reject H_0 . That is, there is evidence that the mean estimate of the facility under suspicion is significantly greater than the mean estimate by the investigators.

Scoring

Part 1 is essentially correct for a correct statement of the hypotheses (in terms of *population* means).

Part 2 is essentially correct if the test is correctly identified by name or formula and a graphical check of the normality condition is given.

Part 3 is essentially correct for a correct calculation of both the test statistic t and the P -value.

Part 4 is essentially correct for a correct conclusion in context, linked to the P -value.

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| 4 Complete Answer | All four parts essentially correct. |
| 3 Substantial Answer | Three parts essentially correct. |
| 2 Developing Answer | Two parts essentially correct. |
| 1 Minimal Answer | One part essentially correct. |

5. (a) A statistic used to estimate a population parameter is unbiased if the mean of the sampling distribution of the statistic is equal to the true value of the parameter being estimated. Estimators B, C, and D appear to have means equal to the population mean of 146.
- (b) For $n = 40$, estimator A exhibits the lowest variability, with a range of only 2 grams compared to the other ranges of 6 grams, 4 grams, 4 grams, and 4 grams.
- (c) The estimator should have a distribution centered at 146, thus eliminating A and E. As n increases, D shows tighter clustering around 146 than does B. Finally, while C looks better than D for $n = 40$, the estimator will be used with $n = 100$, and the D distribution is clearly converging as the sample size increases while the C distribution remains the same. Choose D.

Scoring

Part (a) is essentially correct for a correct answer with a good explanation of what unbiased means, and is partially correct for a correct answer with a weak explanation. Part (a) is incorrect for a correct answer with no explanation or with an incorrect explanation.

Part (b) is essentially correct for a correct answer together with some numerical justification, and is partially correct for a correct answer with a weak explanation. Part (b) is incorrect for a correct answer with no explanation or with an incorrect explanation.

Part (c) is essentially correct for a correct answer with a good explanation, and is partially correct for a correct answer with a weak explanation. Part (c) is incorrect for a correct answer with no explanation or with an incorrect explanation.

4 Complete Answer	All three parts essentially correct.
3 Substantial Answer	Two parts essentially correct and one part partially correct.
2 Developing Answer	Two parts essentially correct OR one part essentially correct and one or two parts partially correct OR all three parts partially correct.
1 Minimal Answer	One part essentially correct OR two parts partially correct.

6. (a) State the hypotheses:

$H_0 : p_H - p_L = 0$ and $H_a : p_H - p_L > 0$ where p_H is the proportion of patients receiving higher doses who experience severe nausea, and p_L is the proportion of patients receiving lower doses who experience severe nausea [other possible expressions include $H_0 : p_H = p_L$ and $H_a : p_H > p_L$].

Identify the test by name or formula and check the assumptions:

Two-sample test for proportions

$$z = \frac{\hat{p}_H - \hat{p}_L}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_H} + \frac{1}{n_L}\right)}}$$

Assumptions: Patients were randomly placed in different groups, so it is reasonable to assume independence of samples. Then we note that

with $\hat{p}_H = \frac{205}{405} = .506$ and $\hat{p}_L = \frac{88}{395} = .223$, we have $n_H\hat{p}_H = 205$,

$n_H(1 - \hat{p}_H) = 200$, $n_L\hat{p}_L = 88$, and $n_L(1 - \hat{p}_L) = 307$. These are all greater than 10.

Calculate the test statistic and the P -value:

$$\hat{p} = \frac{205 + 88}{405 + 395} = \frac{293}{800} = .366$$

$$z = \frac{.506 - .223}{\sqrt{(.366)(.634)\left(\frac{1}{405} + \frac{1}{395}\right)}} = \frac{.283}{.034} = 8.32$$

$$P\text{-value} = .000$$

[Note: a graphing calculator will give $z = 8.31792$ with $P = 4.5279\text{E-}17$]

State the conclusion in context with linkage to the P -value:

With this small a P -value ($P < .001$), there is very strong evidence to reject H_0 . That is, there is very strong evidence that a greater proportion of patients receiving higher doses experience severe nausea than patients receiving lower doses.

(b) Identify the confidence interval by name or formula:

95% confidence interval for the slope of the regression line $b \pm ts_b$

Check the assumptions:

The scatterplot is roughly linear, there is no apparent pattern in the residuals plot, and the distribution of the residuals is approximately normal (because the normal probability plot is roughly linear).

Calculate the confidence interval:

$$df = n - 2 = 8 - 2 = 6$$

$$0.006659 \pm 2.447(0.001761) = 0.006659 \pm 0.004309$$

$$(0.00235, 0.010968)$$

Interpret the confidence interval in context:

We are 95% confident that the mean proportion of patients experiencing severe nausea goes up between 0.00235 and 0.010968 with each increase of 1 milligram per week in dose intensity.

(c) The probability that a randomly chosen patient experienced severe nausea

is $\frac{293}{800} = .366$, so the probability that at least 3 out of 5 experienced severe

nausea is $\binom{5}{3}(.366)^3(.634)^2 + \binom{5}{4}(.366)^4(.634) + \binom{5}{5}(.366)^5 = 10(.366)^3(.634)^2$

$+ 5(.366)^4(.634) + (.366)^5 = .261$ [Or $1 - \text{binomcdf}(5, .366, 2)$]

Scoring

Part (a) has four parts: 1) stating the hypotheses; 2) identifying the test and checking assumptions; 3) calculating the test statistic and the P -value; and 4) giving a conclusion in context with linkage to the P -value. Part (a) is essentially correct if three or four of these parts are correct and partially correct if one or two of these parts are correct.

Part (b) has four parts: 1) identifying the confidence interval; 2) checking assumptions; 3) calculating the confidence interval; and 4) interpreting the confidence interval in context. Part (b) is essentially correct if three or four of these parts are correct and partially correct if one or two of these parts are correct.

Part (c) is essentially correct if the correct probability is calculated and the derivation is clear. Part (c) is partially correct for indicating a binomial with $n = 5$ and $p = .366$, but then calculating incorrectly.

4 Complete Answer	All three parts essentially correct.
3 Substantial Answer	Two parts essentially correct and one part partially correct.
2 Developing Answer	Two parts essentially correct OR one part essentially correct and one or two parts partially correct OR all three parts partially correct.
1 Minimal Answer	One part essentially correct OR two parts partially correct.