Name: MR. YOUNG - Rubric Date: 1/12/2014 Prd 1, 2, 3

SEQUENCES AND SERIES: REVIEW PACKET!

Arithmetic/Geometric Sequences, Summation Notation, Recursive Sequences, Partial/Infinite Sums, Binomial Expansions

Part 1: Find the first four terms as well as the tenth term of the sequence. Then find the 4th partial sum.

1.) $a_n = \frac{n^2}{n+1}$

$a_1 = \frac{1^2}{1+1} = \frac{1}{2}$

$a_2 = \frac{2^2}{2+1} = \frac{4}{3}$

$a_3 = \frac{3^2}{3+1} = \frac{9}{4}$

$a_4 = \frac{4^2}{4+1} = \frac{16}{5}$

$a_{10} = \frac{10^2}{10+1} = \frac{100}{11}$

$S_4 = \frac{1}{2} + \frac{4}{3} + \frac{9}{4} + \frac{16}{5}$

$= \frac{30}{60} + \frac{80}{60} + \frac{135}{60} + \frac{192}{60}$

$\boxed{S_4 = \frac{437}{60}}$

2.) $a_n = \frac{n(n+1)}{2}$

$a_1 = \frac{1(1+1)}{2} = 1$

$a_2 = \frac{2(2+1)}{2} = 3$

$a_3 = \frac{3(3+1)}{2} = 6$

$a_4 = \frac{4(4+1)}{2} = 10$

$a_{10} = \frac{10(11)}{2} = 55$

$S_4 = 1+3+6+10$

$\boxed{S_4 = 20}$

Part 2: A sequence is defined recursively. Find the first 3 terms and the 10th term of sequence (use your calculator to find the 10th term).

3.) $a_n = \frac{a_{n-1}}{n}, \quad a_1 = 1$

$a_1 = 1$

$a_2 = \frac{1}{2}$

$a_3 = \frac{\frac{1}{2}}{3} = \frac{1}{6}$

$\boxed{a_{10} = 2.8 \times 10^{-7}}$

Part 3: SIGMA NOTATION: Write the sum using sigma notation. Do Not Evaluate.

4.) 3 + 6 + 9 + 12 + ... + 99

$\boxed{\sum_{n=1}^{33} 3n}$

5.) $1^2 + 2^2 + 3^2 + \ldots 100^2$

$\boxed{\sum_{n=1}^{100} n^2}$

Write the sum WITHOUT sigma notation. Do Not Evaluate. (you may use ...)

6. $\sum_{k=1}^{10}(k-1)^2$

$(1-1)^2 + (2-1)^2 + (3-1)^2 + \ldots (10-1)^2$

$= \boxed{0^2 + 1^2 + 2^2 + \ldots 9^2}$

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Part 4: Tell whether the sequence is arithmetic, geometric or neither. Explain.

7.) 5, 9, 13, 17, ...

+4 +4 +4

Arithmetic

diff. btw consecutive terms is constant $(d=4)$

$(+4)$

8.) 3, 6, 12, 24, ...

×2 ×2 ×2

Geometric:

Increases by a common ratio r. $(r=2)$

9.) 40, 10, $\frac{5}{2}$, $\frac{5}{8}$, ...

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

Geometric.

each subsequent term is 1/4 the previous term.

$r=\frac{1}{4}$.

Part 5: Write the next term of the sequence, and then write a rule for the nth term.

10.) 5, 11, 17, 23,... $\boxed{29}$

$d=6$

$a_1=5$

$a_n = a_1 + (n-1)d$

$a_n = 5 + (n-1)6$

$\boxed{a_n = 5 + (n-1)6}$

OR

$\boxed{a_n = -1 + 6n}$

11.) $\frac{6}{5}, \frac{7}{10}, \frac{8}{15}, \frac{9}{20}, \ldots \frac{10}{25}$

$\boxed{a_n = \frac{n+5}{5n}}$

Part 6: Find the first four partial sums and the nth partial sum of the sequence a_n.

12.) $a_n = \frac{3}{4^n}$

$S_1 = \frac{3}{4^1} = \boxed{3/4}$

$S_2 = \frac{3}{4^1} + \frac{3}{4^2} = \boxed{15/16}$

$S_3 = \frac{3}{4} + \frac{3}{4^2} + \frac{3}{4^3} = \boxed{63/64}$

$S_4 = \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \frac{3}{256} = \boxed{255/256}$

$\boxed{S_n = \frac{4^n - 1}{4^n}}$ or $\boxed{1 - \frac{1}{4^n}}$

Part 7: Determine if the series converges. If it does, find the sum of the series.

13.) $1 + 0.9 + 0.9^2 + \ldots$

0.9 0.9

$r = 0.9$

$a_1 = 1$

Converges because

$|r| < 1$

$|0.9| < 1$

$= \frac{a_1}{1-r}$

$= \frac{1}{1-0.9} = \frac{1}{0.1} = \boxed{10}$

Sum of the infinite series.

14.) $\sum_{n=1}^{\infty} 5 \bullet 1.02^{n-1}$

5, 5.1, 5.202, 5.306...

$|r| \geq 1$

series will diverge (grows without bound)

$\boxed{\text{Divergent}}$

i.e. Infinite sum is ∞

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15.) *Find the sum of the ~~geometric~~ series (~~don't just use your calculator~~)*

a.)

$$\sum_{n=1}^{n=10} 5 \cdot 2.5^{n-1} = 5, 12.5, 31.25 \ldots 19{,}073.49$$

$r = 2.5$

$a_1 = 5 \cdot 1 = 5$

Partial sum of a geometric sequence: formula

$$\boxed{\frac{a_1(1-r^n)}{1-r} = S_n}$$

sum

$$S_{10} = \frac{5(1-2.5^{10})}{(1-2.5)} = \boxed{31{,}785.8}$$

b.)

$$\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n = 2, \frac{4}{3}, \frac{8}{9}, \ldots$$

$r = 2/3$ $\quad$ $2/3$

$|r| < 1$ converges

$$\frac{2}{2 - 2/3} = \frac{2}{\frac{1}{3}} = \boxed{6}$$

Part 8: Word Problems

16.) Stephen DeLeo drops a lacrosse ball from Friends Hall from a height of 100 feet. The elasticity of the ball is such that it always bounces up 2/5 the distance it has fallen. Find the total vertical distance the ball will travel until it stops (assuming it doesn't hit ~~Spencer Greer~~ or ~~Tristan~~ in the head).

BRAD IGGY

100ft

$$2 \sum_{n=1}^{\infty} 100\left(2/5\right)^{n-1}$$

height of ball after each bounce

$100 + 40 + 16 + 6.4 \ldots$

$|r| < 1$ so $\frac{a_1}{1-r}$

$$\frac{100}{1 - 2/5} = \frac{100}{3/5} = 166.67$$

multiply by 2 as the vert distance is the up AND down

$$2 \cdot 166.67$$
$$- 100$$
$$\boxed{233.33\text{ft}}$$

subtract 100 because the first drop only has a down so shouldn't have been doubled.

17.) The Bank of America Pavilion in Boston has seating where the number of seats per row increases as you get further away from the stage. You are bored at the Taylor Swift concert ("no way Mr. Young!") and decide to figure out the number of seats that the pavilion has. The pavilion has 130 rows of seats with 40 seats in the first row, 42 in the second, 44 in the third and so on. Find the total number of seats in the pavilion. Show your work.

$d = 2$

$a_1 = 40$

$40, 42, 44 \ldots$ $\quad$ $+2 \quad +2$

$a_n = a_1 + (n-1)d$

$a_{130} = 40 + (130-1)2$

$a_{130} = 298$ ← 298 seats in the last row.

Sum of an arithmetic sequence

$$(a_1 + a_n)\frac{n}{2}$$

$$(40 + 298)\frac{130}{2}$$

$$= \boxed{21{,}970 \text{ total seats}}$$

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18.) At the age of 21, Cyndi began receiving yearly payments from a trust fund. On each succeeding birthday, she received twice as much as in the preceding year. If she had received a **total** of $303,800 by age 25, how much did she receive at age 21 (her first year of receiving trust money)?

$a_1 = ?$

$r = 2$

Geometric series

5th partial sum

Solve for a_1

$$303,800 = \frac{a_1(1-2^5)}{1-2}$$

$a_5 = 303,800$

Formula for Partial sum of a geom. seq.

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$-303,800 = a_1(1-2^5)$$

$$\frac{-303,800}{-31} = \frac{-31a_1}{-31}$$

$\boxed{a_1 = \$9,800}$

Check:
9800 + 19,600 + 39,200 + 78,400 + 156,800 = ✓303,800

Part 7: Binomial Expansions:

19.) Expand the binomial (you may use Pascal's triangle or the binomial theorem). $(2x + y)^6$

PASCAL'S TRIANGLE

0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1

$a = 2x$

$b = y$

$$1a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

$$(2x)^6 + 6(2x)^5(y) + 15(2x)^4(y^2) + 20(2x)^3(y^3) + 15(2x)^2(y^4) + 6(2x)(y^5) + y^6$$

$$\boxed{64x^6 + 192x^5y + 240x^4y^2 + 160x^3y^3 + 60x^2y^4 + 12xy^5 + y^6}$$

OR by the binom. theorem (though Pascal's Δ is faster here)

$$\binom{6}{0}a^6 + \binom{6}{1}a^5b^1 + \binom{6}{2}a^4b^2 \ldots$$

20.) Find the term containing A^6 in the expansion of $(A + 3B)^{10}$

$n = 10$

$r = 6$

Formula

$$\binom{n}{n-r}a^r b^{n-r}$$

$a = A$

$b = 3B$

$$\binom{10}{6}a^6 b^{10-6}$$

★ Here's where the binomial theorem is useful.

$$210\,(A)^6(3B)^4$$

$$210\,A^6\,(81B^4)$$

$$\boxed{17,010\,A^6B^4}$$