Name: Mr. Young - Solutions Date: 11/26/2013 Prd: 1,2,3

HONORS PRECALCULUS: STATISTICS PROBLEM SET

HW #36 (worth 25 points)

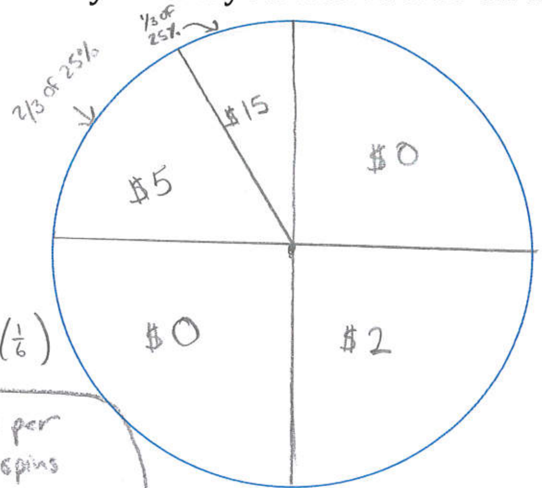
Show ALL work and box answers!

Part 1: Expected Value

1.) Bruce and Stephen have created the following game below. It costs \$2 to spin and play (you lose your 2 dollars no matter what). What is the expected value of a spin? If you played 1000 times in a row, how much money would you win or lose on average.

$$E(x) = \sum x \cdot p(x)$$

Outcome	\$5	\$0	\$15	\$2	\$-2
Prob:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{4}$	1



$$E(x) = -2(1) + 2\left(\tfrac{1}{4}\right) + 15\left(\tfrac{1}{12}\right) + 0\left(\tfrac{1}{2}\right) + 5\left(\tfrac{1}{6}\right)$$

$$\boxed{E(x) = \$0.58}$$

Average 58 cents per spin. So 1000 spins would yield \$580 on average.

2.) Amelia, Nicole, and Caitlin are sitting at the Roulette (much to the surprise of their math teacher and parents...)

Nicole's is pretty sure that Red is lucky tonight. What is the expected value of a \$5 bet on red?

Another way to think about it: $\left(\frac{18}{38}\right)(10) + (1)(-5) = -0.26$

$$E(x) = \frac{18}{38}(5) + \frac{20}{38}(-5)$$

$$\boxed{E(x) = \$-0.26}$$ ← Expect to lose 26 cents per \$5 bet, on average.

Amelia and Caitlin disagree. Even and 12 are looking favorable. What is the total expected value of a \$10 bet on 12 and a \$5 bet on EVEN?

Another Method: $\frac{1}{38}(360) + (1)(-10) + \frac{18}{38}(10) + (1)(-5) = -0.79$

$$E(x) = \underbrace{\frac{1}{38}(350) + \frac{37}{38}(-10)}_{\$10 \text{ bet on } 12} + \underbrace{\frac{18}{38}(5) + \frac{20}{38}(-5)}_{\$5 \text{ bet on odd}}$$

$$\boxed{E(x) = \$-0.79}$$

Remember that you will double your money for a bet on even (i.e. if you bet a dollar and it lands on even, then you will make another \$1). For a bet on a single number, if it lands on the number then you will make 35 times what you put in. So for a 1 dollar bet, the casino will pay you \$35.

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Part 2: Permutations and Combinations

1.) How many different ways are there to arrange the letters in the word MEGAN?

$5 \, nPr \, 5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ 5!

$= \boxed{120 \text{ ways}}$

2.) How about the word AMEERA? Careful here!

2A'S
2E'S

$= \frac{6!}{2!2!} = \frac{720}{2 \cdot 2} = \frac{720}{4} = \boxed{180 \text{ ways}}$

3.) Hayden, Keenan, Aidan, Brad, Max, and Owen are playing cards (7-card stud poker) with a 52 card deck. How many different 7-card hands are possible?

ORDER DOES NOT MATTER

$52 \, nCr \, 7 = \frac{52!}{7!(52-7)!} = \boxed{133{,}784{,}560 \text{ WAYS}}$

4.) Oliver is planning a trip to the DR this Spring with MB students. Christina Mouradian highly recommends going!! 30 people have applied and there are 12 spots available. How many ways are there to pick a group of 12 people to go on the trip to the Dominican Republic from a group of 30 people?

$30 \, nCr \, 12 = \frac{30!}{12!(30-12)!} = \boxed{86{,}493{,}225 \text{ ways}}$

5.) Olivia and Cat Mazo are designing a new type of soccer cleat that makes you incredibly FAST. They want a way to label the shoes that they manufacture so that they can identify each cleat if there is a customer concern. They decide to label each cleat with 3 different letters (which does not include the letters I (i), O (o), or Q (q) (to avoid confusion with numerals 1 and 0). followed by 4 different numbers. How many cleats can they produce with unique identifying codes?

26 − 3 = 23 letters
10 digits

$\underbrace{\boxed{23} \cdot \boxed{22} \cdot \boxed{21}}_{\text{letters}} \cdot \underbrace{\boxed{10} \cdot \boxed{9} \cdot \boxed{8} \cdot \boxed{7}}_{\text{digits}} = \boxed{53{,}555{,}040 \text{ Codes}}$

OR (if you prefer)

$(23 \, nPr \, 3)(10 \, nPr \, 4) =$

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6.) Alex R. and Alex I. and have proposed that Moses Brown have a daily lottery to raise money for the sports dome it wants to build (that would be pretty controversial I think...). To win the lottery they will pick 4 numbers from a bin of 20 WHITE balls (no repetitions) and 1 NAVY Ball from a bin of 18 NAVY balls. To win the jackpot you must match the 4 WHITE balls (in any order) and the NAVY ball.

What are the odds of winning the jackpot?

$$20 \text{ nCr } 4 = \frac{20!}{4!(20-4)!} = 4895 \cdot 18 = 87,210 \qquad \boxed{\frac{1}{87,210}}$$

What is the expected value of a ticket that costs $1 if the jackpot is currently $150,000?

$$\left(\frac{1}{87,210}\right)(150,000) + (1)(-1) = \boxed{\$0.72}$$

← These tickets are a good deal!

Gabby, Eleanor, and Colette think the game is a sham. How much should the ticket cost for the game to have an expected value of zero?

cost of ticket ↓

$$E(x) = \left(\frac{1}{87,210}\right)(150,000) - x$$

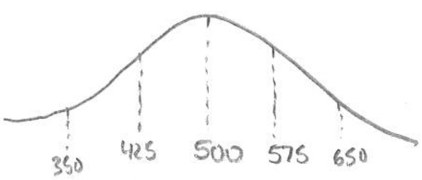
$$\boxed{x = \$1.72}$$

← The tix. should cost $1.72

Part 3: The Normal Distribution/Z-Table

1.) Ananya and Lauren are crunching some numbers. The SAT Verbal in the country is normally distributed and has a mean of 500 and a standard deviation of 75. If you scored a 650 on the SAT VERBAL, DRAW A SKETCH OF THE NORMAL DISTRIBUTION WITH THE MEAN AND 2 STD. DEVIATIONS TO THE LEFT AND RIGHT.

a.) what percent of test takers scored worse than you?



$$z = \frac{650 - 500}{75} = 2$$

from z-table $\boxed{97.72\%}$

b.) If you score a 410, what percentile are you in?

$$z = \frac{410 - 500}{75} = \frac{-90}{75} = -1.2$$

z-table ↓ 0.1150

$\boxed{11.5\%}$

c.) What percent of test takers score between a 550 and 650?

97.72 − 74.86

$$\frac{550-500}{75} = 0.67 \qquad \frac{650-500}{75} = 2$$

74.86% 97.72%

$= \boxed{22.86\%}$

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Part 4: Confidence Intervals

1.) Kara, Lydia, and Reed decide to do a study to see if students enjoy Meeting for Worship at Moses Brown. They conduct an SRS of 45 students and 31 said yes, they like going to MFW. What can the researchers conclude from the survey? a.) Find a 95% confidence interval for the percent of students that like the lunches at Moses Brown)

$\hat{p} = \frac{31}{45} = 68.89\%$

$0.6889 \pm 1.96 \cdot \sqrt{\frac{0.6889(1-0.6889)}{45}}$

0.6889 ± 0.135

$[0.5539, 0.8242]$

b.) Interpret Your Results: We are 95% confident that the true percent of students who enjoy MFW is btw 55.89% and 82.42%

c.) What would have happened to the interval if the researchers had instead wanted a 99% confidence interval (you don't have to give the specific change just what would generally change).

The interval would be greater (this is because the Zcrit would be 2.576).
The more conf. you want to be, the more you have to extend the interval.

2.) Alfie and Lily want to find the average height of adult residents in Providence. To do so they takes a simple random sample of 375 residents and measures their height. The average height of these residents is
x = 68.7 inches with a standard deviation of s = 3.40 inches.. It is known that the heights of these residents is normally distributed.

a.) Find a 95% confidence interval for the average height (μ) of Providence residents? What is the Margin of Error?

$\bar{x} = 68.7$

$s_x = 3.40$

$n = 375$

$t_{crit} =$

$68.7 \pm 1.984 \cdot \frac{3.40}{\sqrt{375}}$

M.O.E.

from e-table

$[68.355, 69.045]$

b.) Interpret Your Results: We are 95% confident that the true mean height of Providence residents lies between 68.355 and 69.045 inches

Part 5: Hypothesis testing

1.) David and Matt are sick of Mr. Young's batteries running low on his Eno board pen. They have thus developed a new, long lasting lithium battery. They claim that the battery will run continuously for 1300 hours. Mr. Young is skeptical and takes a simple random sample of 30 batteries to be tested. The batteries run for an average of 1150 hours, with a standard deviation of 54 hours. Test the null hypothesis that the mean time is 1300 hours against the alternative hypothesis that the mean run time is less than 1300 hours. Use a 0.05 level of significance. (Assume that run times for the population of batteries are normally distributed.)

$H_0: \mu = 1300$

$H_A: \mu < 1300$

$\bar{x} = 1150$

$S_x = 54$

$n = 30$

$t = \frac{1150 - 1300}{54/\sqrt{30}} = -15.21$

P-value ≈ 0

from calc. or table

We can reject the null hypothesis and can conclude that the mean runtime of the batteries is less than 1300 hours

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2.) Dr. Taylor and Dr. Rao are testing a new drug that lowers cholesterol. It is known that 47% of patients who take the old drug see a lowered rate of cholesterol. Suppose that the doctors in various hospitals have given the new drug to a total of 85 people and that 44 patients see lowered cholesterol levels. Can we justify the claim that the new drug is **better** than the old one (use a 1% level of significance).

H_0: $p = 0.47$

H_1: $p > 0.47$

$\hat{p} = 44/85 = 0.5176$

$n = 85$

$$z = \frac{0.5176 - 0.47}{\sqrt{\frac{0.47(1-0.47)}{85}}} = 0.88$$

from calc. (or z-table)

$\boxed{\text{P-value} = 0.1894}$

INTERPRETATION: We cannot reject the null. There is not enough evidence to suggest that the new drug is better (though with more samples we might be able to justify the claim)

3.) Buonanno, Marzec, McCahan, and Pinsky Industries (BMMPI), a tire manufacturer is considering a newly designed tread pattern for its new winter tires. Tests have indicated that these tires will provide better traction and longer tread life. The last remaining test is for braking effectiveness. The company hopes that the the tire will allow a car travelling at 60 mph to come to a complete stop within an average of 100 feet after the brakes are applied (since they are being applied in the snow). They will adopt the new tread pattern unless there is strong evidence that the tires ARE MORE THAN this objective. The distances (in feet) for 11 stops on a test track were 112, 108, 98, 93, 105, 102, 111, 98, 99, 107, and 100.

Should the company adopt the new tread pattern? Test an appropriate hypothesis and state your conclusion. Use a 0.01 level of significance.

H_0: $\mu = 100$

H_a: $\mu > 100$

$\bar{x} = 103$

$S_x = 6.04$

$n = 11$

$t = 1.64$

$\boxed{\text{P-value} = 0.066}$

I computed a t-test on my calculator.

The company could adopt the new tread according to this study. However, the mean stop time was 103 (which is greater than 100). Despite the result not being statistically sign. it might not be wise to adopt the new tread.

4.) Doctor Hardie, Dr. Riva, and Dr. Whelan have just done a study on a new drug that suppresses pain and published a paper citing that their new drug is effective. THey cited a p-value of 0.0012 or 0.12%. Interpret the meaning of this p-value.

The p-value is the probability that we would see results as unusual as we saw, given that the null hypothesis is true (in this case assuming that the drug is not effective).

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Part 6: REVIEW BAYESIAN STATISTICS

1.) Suppose that 3% of all NHL players use steroids (yup, looking at you Orrin and Anton…), that a NHL player who uses steroids tests positive 97% of the time, and that a player who does not use steroids tests positive 3% of the time.

a.) What is the probability that an NHL player who tests positive for steroids actually uses steroids?

	+	-
Steroids (3%)	97%	3%
!Steroids (97%)	3%	97%

$$P(\text{uses ster.} \mid +) = \frac{P(+ \mid \text{uses}) \cdot P(\text{uses})}{P(+)}$$

$$= \frac{0.97 \cdot 0.03}{0.97 \cdot 0.03 + 0.97 \cdot 0.03} = 0.50$$

50% chance that the person who tests pos. actually uses.

b.) What is the probability that a player who tests **negative** does **NOT** use steroids?

$$P(\text{Not Steroids} \mid -) = \frac{P(- \mid \text{not ster.}) \cdot P(\text{not})}{P(-)} = \frac{0.97 \cdot 0.97}{0.97 \cdot 0.97 + 0.03 \cdot 0.03}$$

$= 0.9990$

99.9% chance that a person who tests negative does not use steroids

2.) Agents Strickler, Kagan, and Fay screen BAGS at Philadelphia's International airport for narcotics. 92% of bags that contain a narcotics will be searched. 18% of bags that do not contain narcotics will also be searched. 1 out of every 2000 bags contains narcotics.

a. What is the probability that a bag that is being searched actually contains a Narcotics?
b. What is the probability that a bag that is not searched has narcotics in it?

a) $$P(\text{Narc.} \mid \text{searched}) = \frac{P(S \mid N) \cdot P(N)}{P(S)} = \frac{0.92 \cdot \left(\frac{1}{2000}\right)}{0.92\left(\frac{1}{2000}\right) + 0.18\left(\frac{1999}{2000}\right)} = 0.0025$$

$\boxed{0.25\% \text{ chance}}$

b.) $$P(\text{Narc.} \mid \text{not searched}) = \frac{P(!S \mid N) \cdot P(N)}{P(!S)} = \frac{0.08\,(1/2000)}{0.08(1/2000) + 0.82(1999/2000)}$$

$\boxed{0.00000488}$

almost a zero% chance

EXTRA CREDIT FOR THE QUIZ (1 Extra Point).

In an email, make up your own problem dealing with either hypothesis testing or confidence intervals.

Also, provide the solution. Email me at byoung@mosesbrown.org the question and solution (you may just print it out and write the solution if it's easier, but you must send me the question in an email.

Get Creative!! I will choose my favorite question from the class and if yours is selected you will get 3 BONUS POINTS added to your grade!!! PLUS you made the question which I would think would help in answering it!

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STATISTICS FORMULAS:

BAYES THEOREM

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Z-SCORE

$$z = \frac{x - \mu}{\sigma}$$

Expected VALUE

$$E(x) = \sum x \cdot P(x)$$

CONFIDENCE INTERVALS

Permutations:

$${}_nP_r = \frac{n!}{(n-r)!}$$

PROPORTIONS:

$$\hat{p} \pm z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Combinations:

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$

MEANS:

$$\bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} \pm t \cdot \frac{s}{\sqrt{n}}$$

HYPOTHESIS TESTS:

PROPORTIONS

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

MEANS (z-tests and t-tests)

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

CRITICAL Z-VALUES:

90% Conf: $z = 1.645$

95% Conf: $z = 1.96$

99% conf. $z = 2.576$