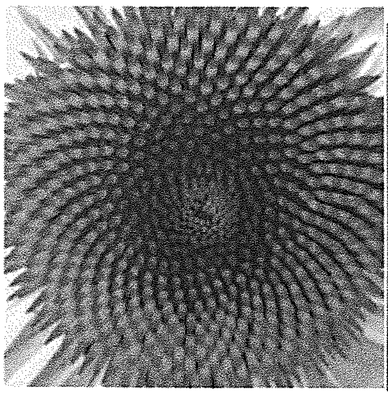
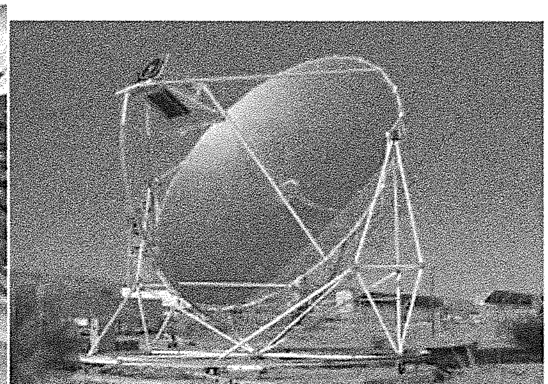
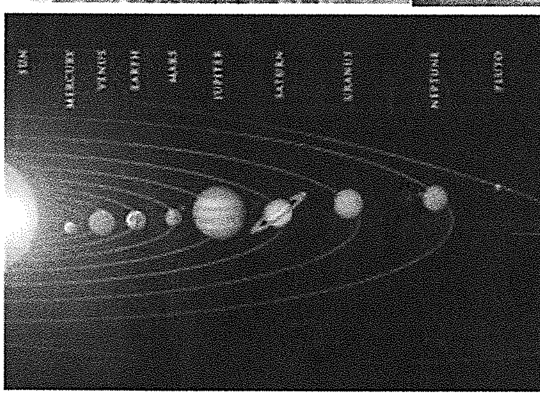
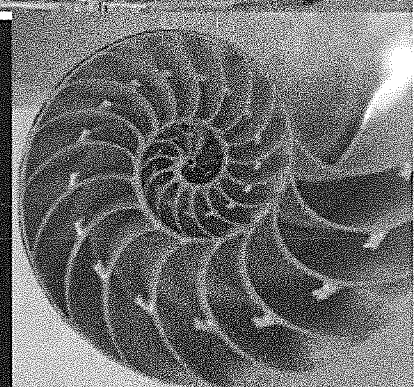
Name: Mr. Young - Rubric Prd:

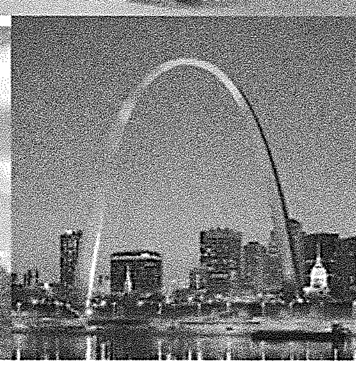
Honors Precalculus

FINAL EXAM (170 points + 3 points E.C.)

June 4th, 2013

"Fear is the path to the dark side. Fear leads to anger, anger leads to hate, hate leads to suffering." ~Yoda



Jeff Cruzan and Ben Young

Directions: Show all work. BOX final answers. All questions are worth 10 points unless indicated otherwise. Good luck!

PART 1: Trigonometry

1.) This summer, you find yourself in the foothills of Switzerland with your family. Eager to put your trigonometry skills to work, you decide that you are going to determine the height of the Matterhorn. You take an angle of elevation to the top of the mountain and find it to be 54°. You then walk 1000 meters closer to the mountain and take an angle elevation to the summit again. This time you find the angle of elevation to be 63.1°. Use trigonometry to determine the height (in meters) of the Matterhorn (Assume you are at sea level and on level ground).

$$\tan(54°) = \frac{h}{x + 1000} \qquad \tan(63.1°) = \frac{h}{x}$$

$$h = \tan(63.1°)\cdot x$$

$$\tan(54°)(x + 1000) = h$$

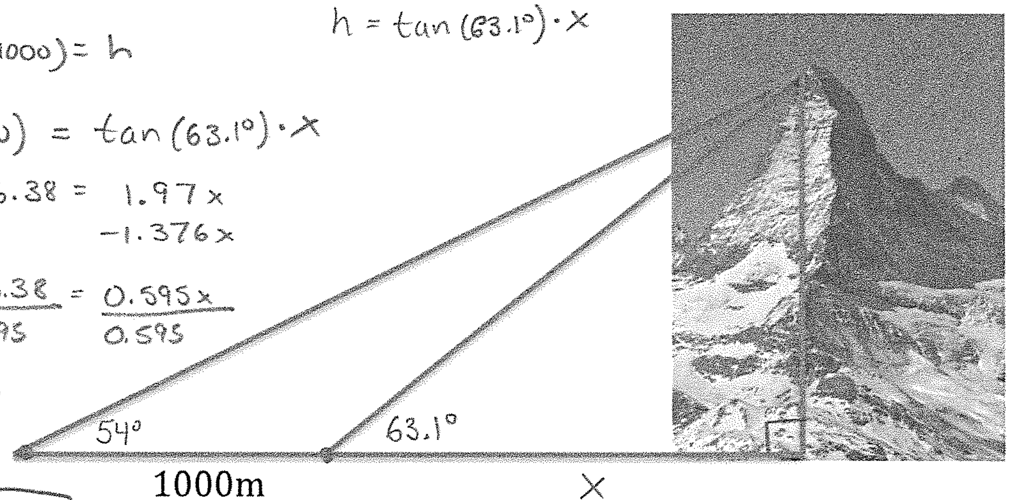
$$\tan(54°)(x+1000) = \tan(63.1°)\cdot x$$

$$1.376x + 1376.38 = 1.97x$$
$$-1.376x \qquad\qquad -1.376x$$

$$\frac{1376.38}{0.595} = \frac{0.595x}{0.595}$$

$x = 2313.24$ m

$\tan(63.1)\cdot 2313.24 = h$



$\boxed{h = 4559.65 \text{ meters}}$

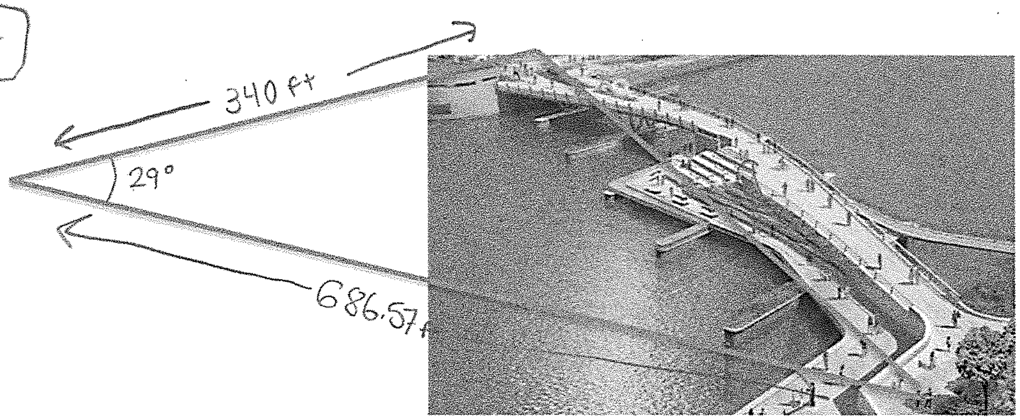
2.) Mayor Taveras is currently in the process of building a new park near downtown Providence (near Brown's Medical School). To increase access to the park, he is building a footbridge across the Providence River going from the Eastside to the park (below is the actual winning design of bridge that they are currently building). As the lead engineer on the project, you have been put in charge of determining the length of the bridge. You take the following measurements using surveying equipment (see diagram below). Use these measurements to determine the length of the bridge.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 340^2 + 686.57^2 - 2(340)(686.57)\cos(29°)$$

$$\sqrt{a^2} = \sqrt{178646.76}$$

$\boxed{a = 422.67 \text{ ft}}$



3.) Verify the identity: $(\sin x + \cos x)^2 = 1 + \sin 2x$

$$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1 + \sin 2x$$

$$\sin^2 x + \cos^2 x + 2\sin x \cos x =$$

$$1 + 2\sin x \cos x =$$

$$1 + \sin(2x) = 1 + \sin(2x) \checkmark$$

4.) Verify the identity: $\frac{\csc\theta+\sec\theta}{\sin\theta+\cos\theta} = \cot\theta + \tan\theta$

$$\frac{\csc\theta+\sec\theta}{\sin\theta+\cos\theta} = \frac{\cos}{\cos}\frac{\cos}{\sin} + \frac{\sin}{\cos}\cdot\frac{\sin}{\sin}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin(\theta)\cos(\theta)}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$\frac{\csc\theta+\sec\theta}{(\sin\theta+\cos\theta)} = \frac{\csc\theta\sec\theta}{1}$$

$$\csc\theta\sec\theta(\sin\theta+\cos\theta) = \csc\theta+\sec\theta$$

$$\frac{1}{\sin}\frac{1}{\cos}\cdot(\sin\theta+\cos\theta) = \csc\theta+\sec\theta$$

$$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \csc\theta+\sec\theta$$

$$\csc\theta+\sec\theta = \csc\theta+\sec\theta$$

5.) Each time your heart beats, your blood pressure increases, then decreases as the heart rests between beats. The average blood pressure for a certain individual is 97 mmHg and the maximum variation from the average is 27 mmHg. The period of the function is 1 second.

a.) Assuming the variation in this individual's blood pressure is simple-harmonic (i.e. sinusoidal), find an equation that models the individual's blood pressure as a function of time (in seconds).

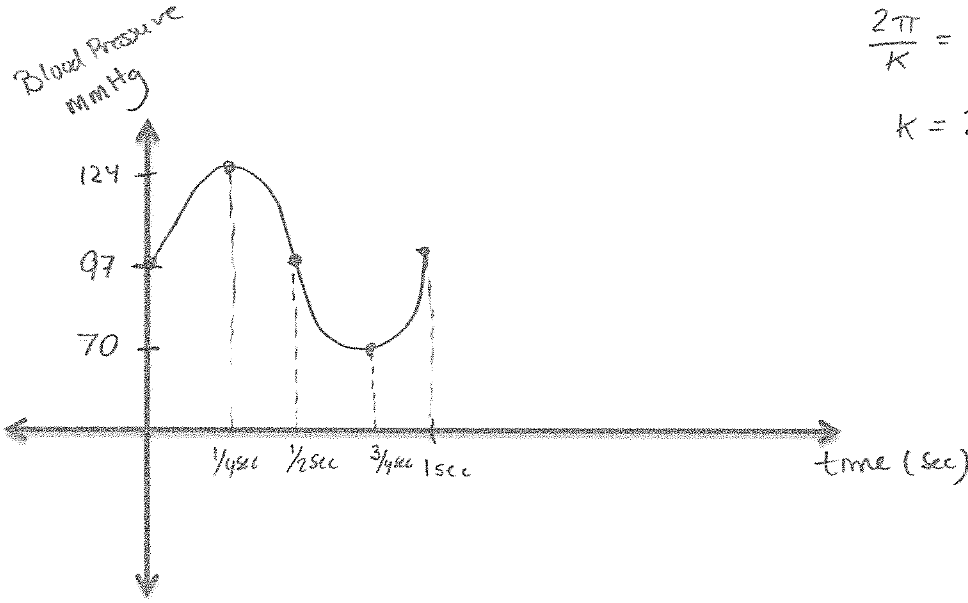
b.) Sketch a graph the function (show one full period)

$\boxed{y = 27(\sin 2\pi x) + 97}$

amp = 27

$\frac{2\pi}{K} = 1$

$K = 2\pi$

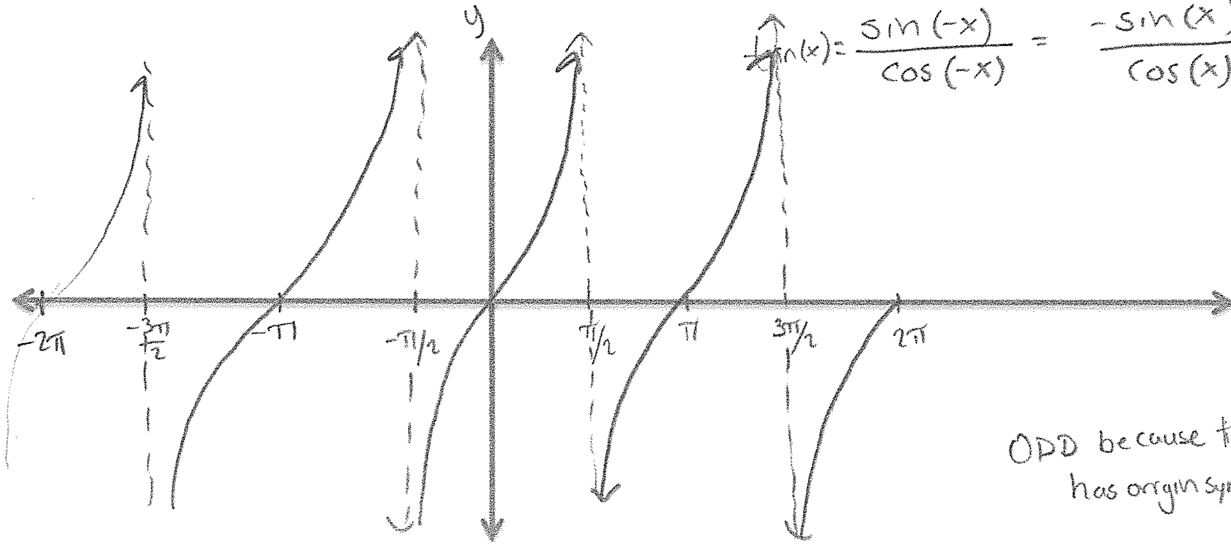


6.) Classify the tangent function as an even or odd function. Explain why using both an equation and a graph (when graphing the tangent function, graph the function on the interval $[-2\pi, 2\pi]$.

tangent is odd:

$\tan(-x) = -\tan(x)$

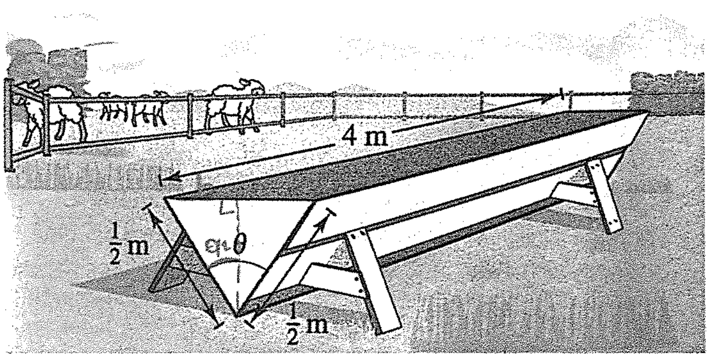
$\tan(x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin(x)}{\cos(x)} = -\frac{\sin(x)}{\cos(x)} = -\tan($



ODD because the function has origin symmetry.

7.) A trough (yup one of these again...) for feeding cattle is 4 meters long and its cross sections are isosceles triangles with two equal sides of $\frac{1}{2}$ meter (see figure below). The angle between the equal sides is θ.

a.) Write the volume of the trough as a function of θ.
b.) Determine the value of θ such that the volume is a maximum. What is the max volume?



a.) $V = \frac{1}{2} b \cdot h \cdot l$

$V = \frac{1}{2} b \cdot h \cdot 4$

$V = 2b \cdot h$

$\sin\left(\frac{\theta}{2}\right) = \frac{\frac{1}{2}b}{\frac{1}{2}}$ $\quad \cancel{2} \cdot \frac{1}{2} b = \frac{1}{2}\sin\left(\frac{\theta}{2}\right) 2$

$b = \sin\left(\frac{\theta}{2}\right)$

$\cos\left(\frac{\theta}{2}\right) = \frac{h}{\frac{1}{2}}$

$h = \cos\left(\frac{\theta}{2}\right) \cdot \frac{1}{2}$

$V = 2 \cdot \sin\left(\frac{\theta}{2}\right) \cdot \frac{1}{2} \cos\left(\frac{\theta}{2}\right)$

$\boxed{V = \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)}$

b.) $\theta = 90° = \pi/2$

$V_{max} = \frac{1}{2} m^3$

8.) Sketch the function $f(\theta) = 3\sin\left(2\theta - \frac{\pi}{2}\right) + 2$ on the interval $[0, 2\pi]$ with θ in radians.

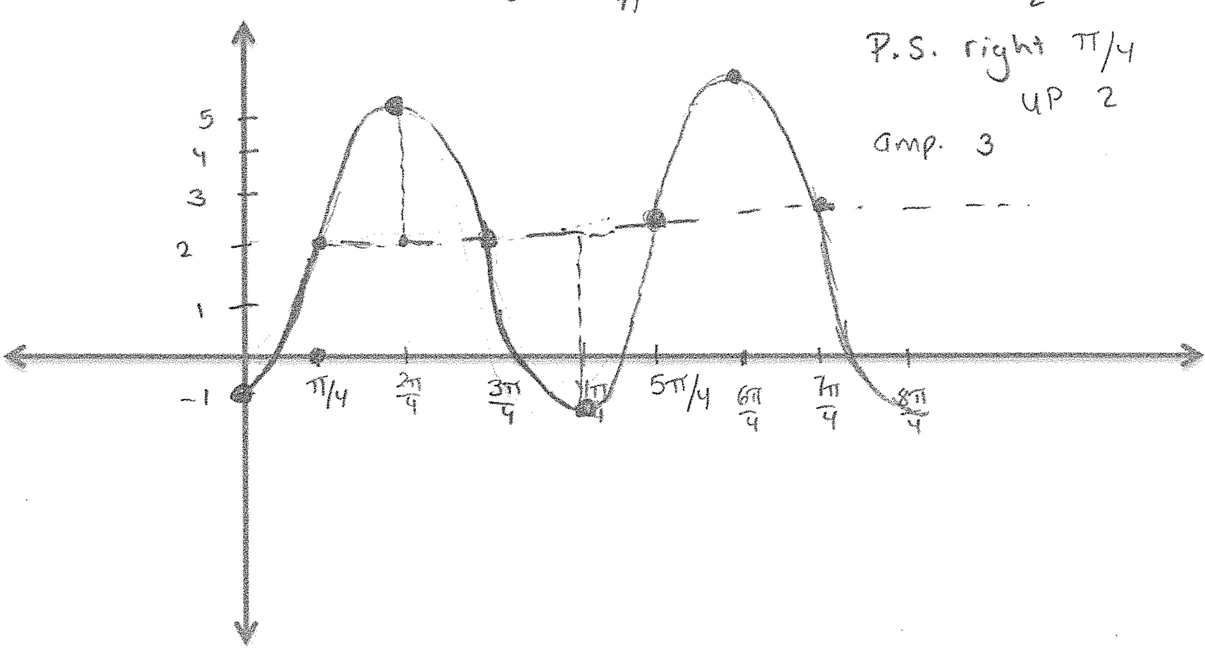
$3 \sin 2\left(\theta - \frac{\pi}{4}\right) + 2$

Period $= \frac{2\pi}{2} = \pi$

P.S. right $\pi/4$

UP 2

amp. 3



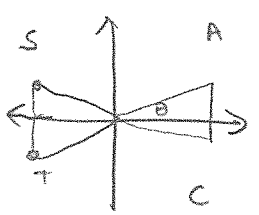
9.) Find all solutions to the equation in the interval $[0, 2\pi)$

$4\,cos^2 x - 3 = 0$

$$\frac{4\cos^2 x}{4} = \frac{3}{4}$$

$$\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}}$$

$$\cos x = \pm\frac{\sqrt{3}}{2}$$



$$x = \pi/6,\ 11\pi/6$$

$$x = \frac{5\pi}{6},\ 7\pi/6$$

$$\boxed{x = \pi/6,\ 5\pi/6,\ 7\pi/6,\ 11\pi/6}$$

Part 2: Converting between polar and rectangular form.

10.) Convert the rectangular equation 3x – 6y + 2 = 0 to polar form and express r as a function of θ.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$3(r\cos\theta) - 6(r\sin\theta) = -2$$

$$\frac{3r(\cos\theta - 2\sin\theta)}{3(\cos\theta - 2\sin\theta)} = \frac{-2}{3(\cos\theta - 2\sin\theta)}$$

$$\boxed{r(\theta) = \frac{-2}{3(\cos\theta - 2\sin\theta)}} = \frac{-2}{3\cos\theta - 6\sin}$$

11.) Convert the polar equation to rectangular form. Also, identify what type of function it represents (linear, circle, ellipse, parabola, hyperbola, etc.)

$r = 6\sin\theta$

$r^2 = 6r\sin\theta$

$x^2 + y^2 = 6y$

$x^2 + y^2 - 6y + 9 = 0 + 9$

$\boxed{x^2 + (y-3)^2 = 9}$

$\boxed{\text{Circle}}$

Part 3: The Complex Plane

12.) (5 points) Find the product $(z_1 z_2)$. Simplify and leave your answer in standard form (a + bi).

$z_1 = 2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$ $z_2 = 8(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6})$

$z_1 \cdot z_2 = 2 \cdot 8\left(\cos\frac{4\pi}{6} + \frac{11\pi}{6} + i\sin\frac{15\pi}{6}\right)$

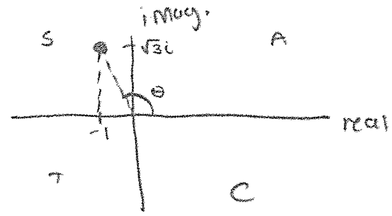
$= 16\left(\cos\frac{15\pi}{6} + i\sin\frac{15\pi}{6}\right)$

$= 16\left(\cos\pi/2 + i\sin\pi/2\right)$

$= 16\left(0 + i(1)\right)$

$\boxed{= 16i}$

13.) Find $\left(-1+\sqrt{3}i\right)^{12}$ using DeMoivre's Theorem:



$r^2 = (\sqrt{3})^2 + (-1)^2$

$r^2 = 3 + 1$

$r^2 = 4$

$r = 2$

$z = 2\left(\cos 2\pi/3 + i \sin 2\pi/3\right)$

$\tan(\theta) = \frac{\sqrt{3}}{-1}$

$z^{12} = 2^{12}\left(\cos \frac{24\pi}{3} + i \sin 24\pi/3\right)$

$\theta = \tan^{-1}(-\sqrt{3})$

$\theta = 2\pi/3$

$z^{12} = 4096\left(\cos 8\pi + i \sin 8\pi\right)$

$= 4096\left(1 + i(0)\right)$

$= \boxed{4096}$

14.) Find the cube roots of 1 (i.e. find ALL the solutions to the equation: $x^3 = 1$). You may leave solutions in polar form.

$r = 1 \quad \theta = 0 \quad \text{or } \theta = 2\pi$

$\sqrt[3]{x^3} = \sqrt[3]{1}$

$x = 1^{1/3}$

$z = 1\left(\cos 0 + i \sin 0\right)$

$a_0 \quad z^{1/3} = 1^{1/3}\left(\cos \frac{0}{3} + i \sin \frac{\theta}{3}\right)$

$\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$

$a_0 = 1(1 + 0)$

$\boxed{a_0 = 1}$

$\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$

$\cos \frac{6\pi}{3} + i \sin \frac{6\pi}{3}$

$a_1 = 1\left(\cos\left(\frac{0+2\pi}{3}\right) + i \sin\left(\frac{0+2\pi}{3}\right)\right)$

$= \boxed{1\left(\cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right)\right)}$

$= -1/2 + i\sqrt{3}/2$

$a_2 = 1\left(\cos\left(\frac{0+4\pi}{3}\right) + i \sin\left(\frac{4\pi}{3}\right)\right)$

$= \boxed{1\left(\cos \frac{4\pi}{3} + i \sin 4\pi/3\right)}$

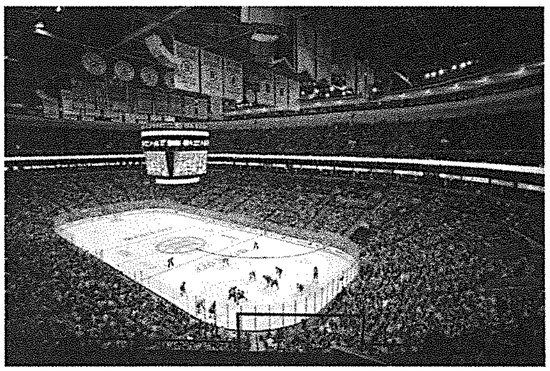
$= -1/2 - (\sqrt{3}/2)$

Part 4: Sequences and Series and Conic Sections

15.) The TD Garden in Boston has seating where the number of seats per row increases as you get further away from the ice. You have arrived 2 hours early to the Bruins game and have decided to figure out how many seats there are in the stadium. The stadium has 75 rows of seats with 150 seats in the first row, 152 in the second, 154 in the third and so on. Find the total number of seats in the stadium (i.e. find the sum of the arithmetic sequence). Show your work.

1	2	3	....	75
150	152	154		

$$\sum_{n=0}^{75} 148 + 2n$$



1st row: 150

75th row: $150 + 2(75-1) = 298$ seats

$$S_{75} = \frac{75}{2}(150 + 298)$$

$S_{75} = 16,800$ seats

16.) Dr. Cruzan drops a golf ball from Friends Hall from a height of 60 feet. The elasticity of the ball is such that it always bounces up 40% the distance it has fallen. Find the total vertical distance the ball will travel until it stops.

$$S = \frac{a}{1-r} = \frac{60}{1-0.40} = \frac{60}{0.60} = 100 \text{ ft.}$$

$$(100\text{ ft})(2) - 60 = \boxed{140 \text{ ft.}}$$

17.) Expand the binomial (you may use Pascal's triangle or the binomial theorem):

$(3x + 2)^4$

$a = 3x$
$b = 2$

$(a+b)^4 =$

$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

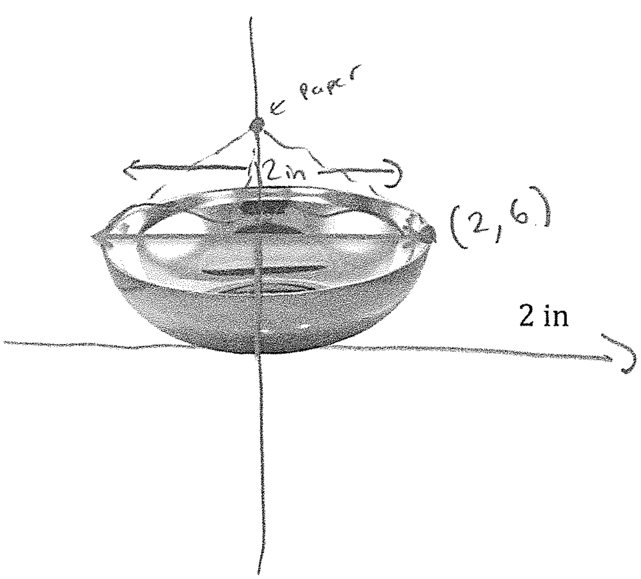
$(3x)^4 + 4(3x)^3(2) + 6(3x)^2(2)^2 + 4(3x)(2)^3 + (2)^4$

$\boxed{81x^4 + 216x^3 + 216x^2 + 96x + 16}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

18.) (5 points) You are stranded in the middle of the mountains and have run out of supplies. Freezing cold and in desperate need of fire you remember the properties of parabolas. You have a reflective aluminum parabolic bowl for eating that you can use to make a parabolic reflector. Being resourceful and remembering everything that you learned in class, you remember that if you reflect the sun off your reflective bowl to the focus of the parabolic bowl that paper will ignite so that you can start a fire.

If you measure the bowl to be 12 inches wide and 2 inches deep, how far above the bottom of the bowl do you need to place the paper in order for it to ignite (i.e. how far is the focus from the bottom of the parabolic bowl)?



$x^2 = 4py$

$6^2 = 4p(2)$

$\frac{36}{8} = \frac{8p}{8}$

$\boxed{\text{P = 4.5 inches above the bottom of the bowl}}$

Extra Credit (3 points) Below is a table showing Usain Bolt's splits in his record breaking 100 meter race where he ran the race in 9.58 seconds (which remains the fastest time ever recorded).

a.) Write a **cubic** regression of the data.

b.) Find the derivative of the cubic regression and use it to determine when Usain Bolt reached his maximum speed. What was his maximum speed in MPH during the race (there are 1609.34 meters per mile)?

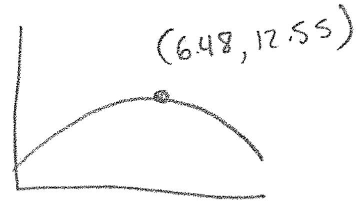
$$\boxed{a.)\ f(x)\ -0.07x^3 + 1.36x^2 + 3.74x - 0.311}$$

b) $f'(x) = -0.21x^2 + 2.72x + 3.74$

$-0.21x^2 + 2.72x + 3.74 = 0$

$x =$

Time (sec.)	Distance
0 s	**0m**
1.85 s	**10m**
2.89 s	**20m**
3.78 s	**30m**
4.64 s	**40m**
5.49 s	**50m**
6.31 s	**60m**
7.11 s	**70m**
7.92 s	**80m**
8.74 s	**90m**
9.58 s	**100m**



after 6.48sec. Bolt reaches his max speed

of 12.55 meters persecond

$\frac{12.55\,m}{\cancel{s}} \cdot \frac{3600\,\cancel{s}}{1\,hr} \cdot \frac{1\,mile}{1609.34} = \boxed{28mp}$