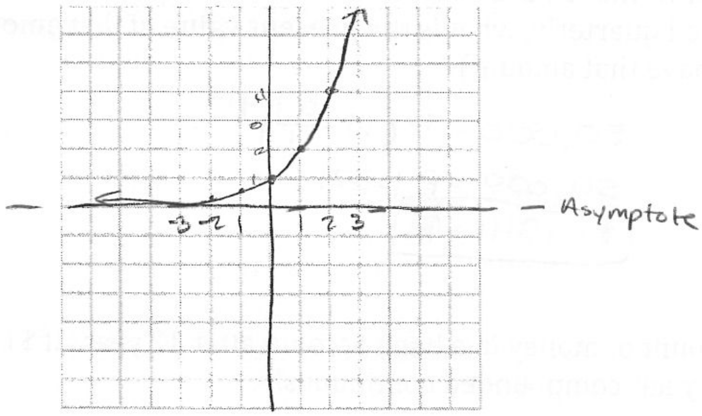
Names: SOLUTIONS and ______________ Date:________

Problem Set: Precalculus (HW 40)

Exponential and Logarithmic Functions

1.) Make a table of values and sketch the graph $f(x) = 2^x$

x	F(x)
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
1	2
2	4



2.) Use the graph of $f(x) = 2^x$ to **describe** the transformation of each.

a.) $g(x) = 3 + 2^x$

UP 3

b.) $h(x) = -2^x$

Flips over x-axis

c.) $k(x) = 2^{x-2} - 3$

down 3
right 2

1.) A certain type of beetle was introduced onto a small island with an initial population of 6 beetles. Scientists estimate that the population of beetles is increasing by 21% every year. a.) Write a function that models the number (n) of beetles after t years.

$\boxed{A = 6(1+.21)^t}$

b.) Create a table showing the number of beetles after 1, 3, 5, 10, 20, 25, and 50 years

$A = 6(1+.21)^1$
$= 7.26$

$A = 6(1+.21)^3$
$A = 10.63$

$A = 6(1+.21)^5$
$A = 15.56$

$A = 6(1.21)^{10}$
$= 40.36$

$A = 6(.21)^{20}$
$= 271.55$

$A = 6(1.21)^{25}$
$= 604.34$

$A = 6(1.21)^{50}$
$= 820083$

FINANCE PROBLEMS

1.) A Sum of a $25,000 is invested into high yield CD (certificate of deposit) which yields 2.5% per year. Find the amounts in the account after 7 years if it is compounded annually, quarterly, or monthly.

$A = P(1+\frac{r}{n})^{n \cdot t}$

$A = 25{,}000(1+\frac{2.5}{1})^7$

$= 29{,}717$

$A = 25{,}000(1+\frac{.025}{4})^{4 \cdot 7}$

$= 29764.94$

$A = 25{,}000(1+\frac{.025}{12})^{12 \cdot 7}$

$A = \$29{,}775$

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2.) Find the **annual percentage yield** of an investment that earns interest at a rate of 0.7% per year, compounded daily.

$A = P\left(1+\frac{0.007}{365}\right)^{365 \cdot 1} = P(1+r)$ simple interest

$\left(1+\frac{.007}{365}\right)^{365} = (1+r)$

$1.00702 = 1+r$ $\boxed{r = 0.702\%}$

$\boxed{.702\%}$

3.) An individual wants to have $50,000 at the end of 7 years. If interest is paid at a rate of 4% per year, compounded quarterly, what is the **present value** of that amount (i.e. what does he need to invest today to have that amount).

$50{,}000 = P\left(1+\frac{.04}{4}\right)^{4 \cdot 7}$

$50{,}000 = P\,1.321$

$\boxed{\$37841 = P}$

4.) Find the amount of money in a bank account after 12 years if $1,000,000 is invested at an interest rate of 12% per year, compounded continuously.

$A = Pe^{rt}$

$A = 1{,}000{,}000\, e^{.12(12)}$

$= \$4{,}220{,}695.81$

5.) The stock market has been growing at a rate of approximately 11.1% per year (over the NYSE's history). If you have $5000 to invest, write a function that models the current value of the investment after t years.

$A = 5{,}000(1+.111)^t$

b.) How much money would you expect to have after 1 year? After 15 years?

$A(1) = \$5555$ $A(15) = \$24248.28$

6.) You have borrowed $4,000 from a bank which charges an interest rate of 5% per year, compounded continuously. How much interest will you owe if you pay back the loan in full 5 years later?

$4{,}000\, e^{.05(5)}$

$\$1{,}136.10$

$5136.10 -$

The Value of E.

1.) Calculate the value of e by hand (either method is fine). Accurate up to 5 decimal places. Show work.

$\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} = 2.71666$

OR

$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ $\left(1+\frac{1}{100{,}000}\right)^{100{,}000} = 2.71826\ldots$

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World Population The relative growth rate of world population has been decreasing steadily in recent years. On the basis of this, some population models predict that world population will eventually stabilize at a level that the planet can support. One such logistic model is

$$P(t) = \frac{73.2}{6.1 + 5.9e^{-0.02t}}$$

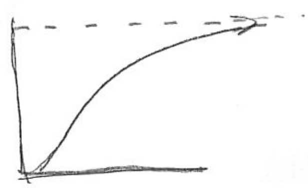
where $t = 0$ is the year 2000 and population is measured in billions.

(a) What world population does this model predict for the year 2200? For 2300?

(b) Sketch a graph of the function P for the years 2000 to 2500.

(c) According to this model, what size does the world population seem to approach as time goes on?

a) 11.791, 11.97
billion people billion people

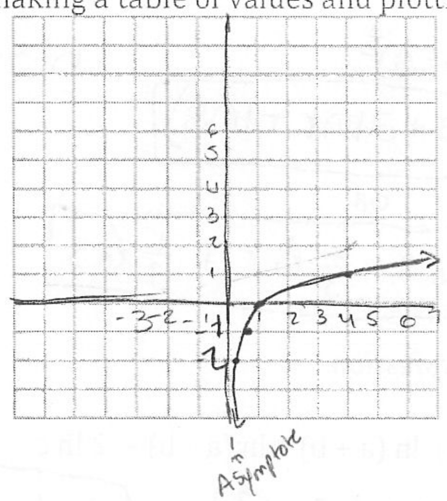
b)


c) 12 billion

Logarithms:

1.) Graph f(x) = log₄x by making a table of values and plotting the points (5 points)

x	F(x)
$\frac{1}{8}$	-2
$\frac{1}{2}$	-1
1	0
4	1
16	2



$4^y = x$

Express the equation in exponential form

2.) a.) $\log_3 81 = 4$ b.) $\ln y = 4$

$3^4 = 81$

$e^4 = y$

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Solve for x. (show your work by writing the log in exponential form or vice versa. Don't simply use a calculator!)

3.) a.) $\log_5 0.2 = x$

$5^x = 0.2$

$\boxed{x = -1}$

b.) $2^{\log_2 37} = x$

$\log_2 x = \log_2 37$

$\boxed{x = 37}$

c.) $\log_5 x = 2$

$5^2 = x$

$\boxed{25 = x}$

LAWS OF LOGARITHMS:

Evaluate the expression using the laws of logs.

1.) $\log_{12}9 + \log_{12}16$

$\log_{12} 144 = \boxed{2}$

2.) $\log_4 16^{100}$

$100 \log_4 16 = \boxed{200}$

Use the Laws of Logarithms to expand the expression

1.) $log_3(x\sqrt{y})$

$\log_3 x + \log_3 y^{\frac{1}{2}}$

$\log_3 x + \frac{1}{2}\log_3 y$

2.) $\ln(\sqrt[3]{3r^2 s})$

$\ln (3r^2 s)^{\frac{1}{3}}$

$\boxed{\frac{1}{3}(\ln 3 + 2\ln r + \ln s)}$

OR

$\boxed{\frac{1}{3}\ln 3 + \frac{2}{3}\ln r + \frac{1}{3}\ln s}$

3.) $\ln\left(\frac{(3x)^2}{(x+1)^{10}}\right)$

$\ln 3 + 2\ln x - 10\ln(x+1)$

Use the Laws of Logarithms to combine the expression

1.) $log\ 12 + \frac{1}{2}log 7 - \log 2$

$\log \frac{(12\sqrt{7})}{2}$

2.) $\ln(a + b) + \ln(a - b) - 2\ln c$

$\frac{\ln(a^2 - b^2)}{\ln c^2} = \boxed{\ln\left(\frac{a^2 - b^2}{c^2}\right)}$

3.) True or False. Does $-\ln\left(\frac{1}{A}\right) = \ln A$. Prove it using the laws of logs.

$-\ln\left(\frac{1}{A}\right)$

$-\ln 1 + \ln A$

$\ln A - \ln 1$

True

$\boxed{\ln \frac{A}{1}}$

OR

$-\ln\left(\frac{1}{A}\right) = -\ln(A^{-1})^{-1} =$

$= \ln(A^{-1})^{-1}$

$= \boxed{\ln A}$ True