Answer Key

1. C	7. E	13. E	19. E	25. C	31. E	37. E
2. D	8. A	14. E	20. A	26. A	32. D	
3. A	9. B	15. C	21. B	27. C	33. C	
4. D	10. B	16. D	22. D	28. E	34. C	
5. E	11. D	17. E	23. A	29. E	35. E	
6. E	12. C	18. E	24. B	30. B	36. E	

Answers Explained

Multiple-Choice

1. **(C)** The critical z-scores will go from ±1.96 to ±2.576, resulting in an increase in the interval size: $\frac{2.576}{1.96} = 1.31$ or an increase of 31%.

2. **(D)** Increasing the sample size by a multiple of d divides the interval estimate by $\sqrt{d}$.

3. **(A)** The margin of error varies directly with the critical z-value and directly with the standard deviation of the sample, but inversely with the square root of the sample size. The value of the sample mean and the population size do not affect the margin of error.

4. **(D)** Although the sample proportion *is* between 77% and 87% (more specifically, it is 82%), this is not the meaning of ±5%. Although the percentage of the entire population is likely to be between 77% and 87%, this is not known for certain.

5. **(E)** The 95% refers to the method: 95% of all intervals obtained by this method will capture the true population parameter. Nothing is certain about any particular set of 20 intervals. For any particular interval, the probability that it captures the true parameter is 1 or 0 depending upon whether the parameter is or isn't in it.

6. **(E)** There is no guarantee that 13.4 is anywhere near the interval, so none of the statements are true.

7. **(E)** The critical z-score with .005 in each tail is 2.576 (from last line on Table B or `invNorm(.005)` on the TI-84), and $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(.11)(.89)}{10{,}000}}$.

8. **(A)** The margin of error has to do with measuring chance variation but has nothing to do with faulty survey design. As long as n is large, s is a reasonable estimate of σ; however, again this is not measured by the margin of error. (With t-scores, there is a correction for using s as an estimate of σ.)

9. **(B)** The critical z-score with .01 in the tails is 2.326 (`invNorm(.01)` on the TI-84), and $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(2/3)(1/3)}{50}}$.

10. **(B)** $\sigma_{\hat{p}} = \sqrt{\frac{(.75)(.25)}{1000}} = .0137$

$.75 \pm 1.96(.0137) = .75 \pm .027$ or $(.723, .777)$

11. **(D)** $\sigma_{\hat{p}} = \sqrt{\frac{(.17)(.83)}{1703}} = .0091$

$z(.0091) = .02 \quad z = 2.20, \quad .9861 - .0139 = 97.2\%$

12. **(C)** $1.645\left(\frac{.5}{\sqrt{n}}\right) \le .04, \sqrt{n} \ge 20.563, n \ge 422.8$, so choose $n = 423$.

13. **(E)** $\frac{19}{20} = 95\%; 1.96\left(\frac{.5}{\sqrt{n}}\right) \le .03, \sqrt{n} \ge 32.67, n \ge 1067.1$, and so the pollsters should have obtained a sample size of at least 1068. (They actually interviewed 1148 people.)

14. **(E)** LOL, OMG, the critical t-score with .02 in the tails is 2.088 (Table B or `invt(.02,79)` on the TI-84), and $\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{15}{\sqrt{80}}$.

15. **(C)** Using t-scores: $28.5 \pm 2.045\left(\frac{1.2}{\sqrt{30}}\right) = 28.5 + 0.45$.

16. **(D)** $\sigma_{\bar{x}} = \frac{1.52}{\sqrt{64}} = 0.19, \frac{0.38}{0.19} = 2$, and $.4772 + .4772 = .9544 \approx 95\%$.

17. **(E)** Using t-scores:

$$335\left[9540 \pm 2.756\left(\frac{1205}{\sqrt{30}}\right)\right] = 335(9540 \pm 606.3) = \$3{,}196{,}000 \pm \$203{,}000.$$

18. **(E)** $1.96\left(\frac{1.1}{\sqrt{n}}\right) \le 0.2, \sqrt{n} \ge 10.78$, and $n \ge 116.2$; choose $n = 117$.

19. **(E)** To divide the interval estimate by d without affecting the confidence level, multiply the sample size by a multiple of d^2. In this case, $4(50) = 200$.

20. **(A)**

$$n_1 = 361 \qquad n_2 = 86$$
$$\hat{p}_1 = \frac{210}{361} = .582 \quad \hat{p}_2 = \frac{34}{86} = .395$$
$$\sigma_d = \sqrt{\frac{(.582)(.418)}{361} + \frac{(.395)(.605)}{86}} = .0588$$
$$(.582 - .395) \pm 1.96(.0588) = .187 \pm .115$$

21. **(B)**

$$n_1 = 300 \quad n_2 = 400$$
$$\hat{p}_1 = .65 \quad \hat{p}_2 = .48$$
$$\sigma_d = \sqrt{\frac{(.65)(.35)}{300} + \frac{(.48)(.52)}{400}} = .0372$$
$$(.65 - .48) \pm 2.576(.0372) = .17 \pm .096$$

22. **(D)** $1.645\left(\frac{.5\sqrt{2}}{\sqrt{n}}\right) \le .03$, $\sqrt{n} \ge 38.77$, and $n \ge 1503.3$; the researcher should choose a sample size of at least 1504.

23. **(A)** $\sigma_d = \sqrt{\frac{(19.1)^2}{347} + \frac{(19.9)^2}{561}}$ and with $df = \min(347 - 1, 561 - 1)$, critical t-scores are ± 1.97.

24. **(B)** $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{(6.3)^2}{274} + \frac{(6.3)^2}{90}} = 0.765$

$(33.0 - 28.6) \pm 1.645(0.765) = 4.4 \pm 1.26$

25. **(C)** $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{(4.5)^2}{n} + \frac{(4.5)^2}{n}} = \frac{6.364}{\sqrt{n}}$

$1.645\left(\frac{6.364}{\sqrt{n}}\right) \le 1$, $\sqrt{n} \ge 10.47$, $n \ge 109.6$; choose $n = 110$.

26. **(A)** The sample mean is at the center of the confidence interval; the lower confidence level corresponds to the narrower interval.

27. **(C)** Narrower intervals result from smaller standard deviations and from larger sample sizes.

28. **(E)** Only III is true. The 90% refers to the method; 90% of all intervals obtained by this method will capture μ. Nothing is sure about any particular set of 100 intervals. For any particular interval, the probability that it captures μ is either 1 or 0 depending on whether μ is or isn't in it.

29. **(E)** In determining confidence intervals, one uses sample statistics to estimate population parameters. If the data are actually the whole population, making an estimate has no meaning.

30. **(B)** $\frac{1.88(.5)\sqrt{2}}{\sqrt{n}} \le .07$ gives $\sqrt{n} \ge 18.99$ and $n \ge 360.7$.

31. **(E)** With $df = 15 - 1 = 14$ and .05 in each tail, the critical t-value is 1.761.

32. **(D)** $\bar{x} = 4.048$, $s = 2.765$, $df = 20$, and

$$4.048 \pm 1.725\left(\frac{2.765}{\sqrt{21}}\right) = 4.048 \pm 1.041.$$

33. **(C)** The confidence interval estimate of the set of nine differences is

$$11.61 \pm 2.306\left(\frac{4.891}{\sqrt{9}}\right) = 11.61 \pm 3.76.$$

34. **(C)** $df = 4$, and $3.2 \pm 2.132\left(\frac{0.274}{\sqrt{5}}\right) = 3.2 \pm 0.261$.

35. **(E)** The critical t-values with $df = 25 - 2 = 23$ are ±2.500. Thus, we have $b_1 \pm t^* \times SE(b_1) = 0.008051 \pm (2.500)(0.001058)$

36. **(E)** The critical t-values with $df = n - 2 = 8$ are ±2.306. Thus, we have $b \pm t^* \times SE(b) = -2.16661 \pm 2.306(1.03092) = -2.16661 \pm 2.3773$ or (–4.54,0.21).

37. **(E)** While there is clearly a positive association between smoking levels and hazard ratios, this was an observational study, not an experiment, so cause and effect (as implied in I, II, and III) is not an appropriate conclusion. The margins of error of the confidence intervals become greater (less precision) with heavier smoking.

Free-Response

1. (a) $\hat{p} = .57$ and $\sigma_{\hat{p}} \approx \sqrt{\frac{(.57)(.43)}{1000}} = .0157$. [Note that $n\hat{p} = 1000(.57) = 570$ and $n\hat{q} = 1000(.43) = 430$ are both greater than 10, we are given an SRS, and 1000 < 10% of all voters.] The critical z-scores associated with the 95% level are ±1.96. Thus the confidence interval estimate is $.57 \pm 1.96(.0157) = .57 \pm .031$, or between 54% and 60%. (We are 95% confident that between 54% and 60% of the voters believe competence is more important than character.)

   (b) Explain to your parents that by using a measurement from a sample we are never able to say *exactly* what a population proportion is; rather we are only able to say we are confident that it is within some range of values, in this case between 54% and 60%.

   (c) In 95% of all possible samples of 1000 voters, the method used gives an estimate that is within three percentage points of the true answer.

2. (a) First, identify the confidence interval and check the conditions:

   This is a one-sample z-interval for the proportion of family members who come down with H1N1 after an initial family member does in this state, that is, $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.