

Exercises

1.39 Phosphate levels The level of various substances in the blood influences our health. Here are measurements of the level of phosphate in the blood of a patient, in milligrams of phosphate per deciliter of blood, made on 6 consecutive visits to a clinic:

5.6 5.2 4.6 4.9 5.7 6.4

A graph of only 6 observations gives little information, so we proceed to compute the mean and standard deviation.

- Find the mean from its definition. That is, find the sum of the 6 observations and divide by 6.
- Find the standard deviation from its definition. That is, find the deviations of each observation from the mean, square the deviations, then obtain the variance and the standard deviation. Example 1.16 shows the method.
- Now enter the data into your calculator to obtain \bar{x} and s . Do the results agree with your hand calculations? Can you find a way to compute the standard deviation without using one-variable statistics?

1.40 Choosing measures of center and spread Which measure of center and spread should be used for the following data sets? In each case, write a sentence or two to explain your reasoning.

- The Treasury bill returns in Figure 1.22(b) (page 88).
- The 60 IQ scores of fifth-grade students in Example 1.6 (page 49).
- The 44 DRP test scores in Exercise 1.5 (page 48).

1.41 Metabolic rates Calculate the mean and standard deviation of the metabolic rates in Example 1.16 (page 85), showing each step in detail. First find the mean \bar{x} by summing the 7 observations and dividing by 7. Then find each of the deviations $x_i - \bar{x}$ and their squares. Check that the deviations have sum 0. Calculate the variance as an average of the squared deviations (remember to divide by $n - 1$). Finally, obtain s as the square root of the variance.

1.42 Median and mean Create a set of 5 positive numbers (repeats allowed) that have median 10 and mean 7. What thought process did you use to create your numbers?

1.43 Contest This is a standard deviation contest. You must choose four numbers from the whole numbers 0 to 10, with repeats allowed.

- Choose four numbers that have the smallest possible standard deviation.
- Choose four numbers that have the largest possible standard deviation.
- Is more than one choice possible in either (a) or (b)? Explain.

CHAPTER

1

Teaching Tip

Emphasize that *both* numerical and graphical information must be considered in order to analyze a data set well.

Answers to Exercises 1.39–1.44

- 1.39** (a) 5.4 mg/dl.
 (b) 0.6419 mg/dl.
 (c) Calculator results should agree with the answers from parts (a) and (b).
- 1.40** (a) The median and *IQR* because the distribution is skewed to the right.
 (b) The mean and standard deviation because the distribution is symmetric.
 (c) The mean and standard deviation because the distribution of DRP scores is roughly symmetric.
- 1.41** $\bar{x} = 1600$ calories; $s^2 = 35812$ squared calories; $s = 189.24$ calories.
- 1.42** Answers will vary. For example: {1, 2, 10, 11, 11}.
- 1.43** (a) Answers will vary. For example: 1, 1, 1, 1.
 (b) 0, 0, 10, 10.
 (c) For (a), any set of the same four numbers will do; (b) is unique.

12. Duri
that
num

Ame
Nat

(a)

1.44 The algebra might be a bit of a stretch for some students:

$$\begin{aligned} & (x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + \\ & (x_{n-1} - \bar{x}) + (x_n - \bar{x}) = \\ & x_1 - \bar{x} + x_2 - \bar{x} \\ & + \cdots + x_{n-1} - \bar{x} + x_n - \bar{x} \\ & \text{(drop the parentheses)} \\ & = x_1 + x_2 + \cdots + x_{n-1} + x_n - \bar{x} - \\ & \bar{x} \cdots - \bar{x} - \bar{x} \end{aligned}$$

(rearrange the terms)

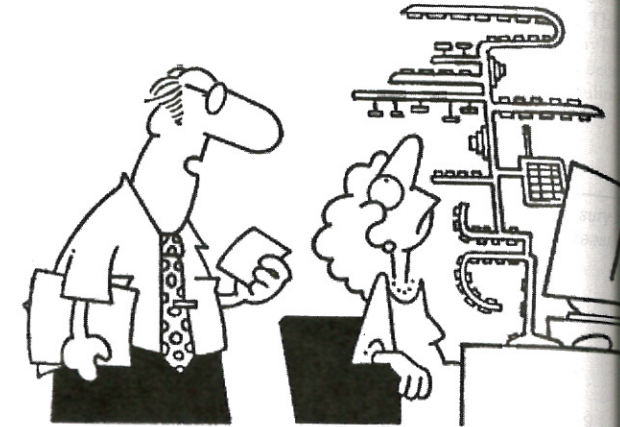
$$= x_1 + x_2 + \cdots + x_{n-1} + x_n - n\bar{x}$$

Next simply observe that $n = x_1 + x_2 + \cdots + x_{n-1} + x_n$.



Statistics in the courtroom

In 1994 Digital Equipment Corporation (DEC) was sued by three individuals who claimed that DEC's computer keyboard caused repetitive stress injuries. Awards for economic loss were fairly easy to set, but deciding awards for pain and suffering was much more difficult. On appeal, Circuit Court Judge Jack Weinstein described ways to find a comparison group of similar cases. Then for the jury award to be considered reasonable, he ruled that it should not be more than two standard deviations away from the mean award of the comparison group. Any award outside this interval would be adjusted to be two standard deviations away from the mean.



"It's the new keyboard for the statistics lab. Once you learn to use it, it will make computation of the standard deviation much easier."

Changing the Unit of Measurement

The same variable can be recorded in different units of measurement. For example, we commonly record distances in miles and temperatures in degrees Fahrenheit, while the rest of the world measures distances in kilometers and temperatures in degrees Celsius. Fortunately, it is easy to convert from one unit of measurement to another. This is true because a change in the measurement unit is a **linear transformation** of the measurements.

Linear Transformation

A **linear transformation** changes the original variable x into the new variable x_{new} given by an equation of the form

$$x_{\text{new}} = a + bx$$

Adding the constant a shifts all values of x upward or downward by a fixed amount. Multiplying by the positive constant b changes the size of the measurement.